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Handbook
Screening test 8+

Contents

I. Introduction

Learning mathematics is cumulative: new content builds on secure prior knowledge. If fundamental ideas and concepts are missing, it becomes increasingly difficult for students to construct meaningful understanding of subsequent topics. Findings from international and national studies show that a substantial share of learners do not have minimum standards in mathematics. For day-to-day teaching this means: early, practical procedures are needed to make learning status visible and to organise timely support. This is where the EU project "Diagnostic Tool in Mathematics (DiToM)" comes in. In a partnership of Italy, France, Sweden, Croatia, Greece, Spain and Germany, five interrelated screening instruments have been developed to provide for teachers a compact overview of their class at educational transition points. The screening points follow a two-year rhythm:

1. Transition pre-school → Start at primary school

2. End of Grade 2 / start of Grade 3

3. End of Grade 4/ start of Grade 5

4. End of Grade 6 / start of Grade 7

5. End of Grade 8 / start of Grade 9

What is a screening?
A screening is a short, group-based assessment that can be administered to the whole class within a single lesson. It provides an initial, structured overview of which core ideas are already solid and where individual students may need additional support. A screening, more importantly, does not replace an individual process-orientated, qualitative assessement of a child's current state of mathematical thinking. It serves as a starting point: results can be followed by targeted observations, interviews and support measures.

Why is this helpful?

- Provide a quick overview: which fundamental skills are secure and where review or extension is useful.

- Allow a guided support: identify students who may struggle with minimum standards of basic mathematics; organise early support.

- Make diagnostic decisions: screening results provide a clear first orientation indicating which students may benefit from further diagnostic steps (e.g., deeper task analyses or follow-up interviews).

- Support transitions: focus attention on key skills at crucial school transitions.

The tasks are classroom-oriented, administration is clearly described and scoring is swift. Teachers receive a concise class-level summary plus pointers to which students merit a closer look for specific content areas. On this basis, they can plan short review windows, differentiated practice or bridging tasks.

This handbook provides a compact guide to the purpose and use of the screening instrument, explains the test design, task types and the targeted assessment objectives, gives clear instructions for classroom administration, outlines scoring and interpretation of results and offers practical ideas for subsequent instruction and targeted support.
The aim is a practical, reliable and easy-to-use screening tool that gives teachers quick orientation, draws early attention to potential difficulties and concretely supports effective help so that as many students as possible learn mathematics securely, with understanding and with confidence.

II. What means "mathematical key skills"

Diagnostic test development requires a theoretical basis. For short, whole-class screening tests, this means focusing on those skills without which subsequent content cannot be learned in a meaning-oriented way. Following the classical position of Gagné & Briggs, each new learning demand builds upon a minimal amount of necessary prerequisites what the term mathematical key skills refer to. If these are not available, successful acquisition of the new content is unlikely and appropriate tasks therefore build on what is already in place. In mathematics, learning is accordingly hierarchical and cumulative.

Conceptual understanding: competences, concepts, skills and key skills

Within the project we distinguish between competences and skills, which are mutually dependent in classroom practice. Competences refer to an insightful readiness to act appropriately in mathematical situations. Thereby, concepts capture substantive insight into mathematical relations. The understandable activation of competencies arise in a skill, as practiced performance on side of the students. Key skills are those skills whose absence substantially hinders or prevents further learning. They function as necessary prerequisites for subsequent content. The focus of the screenings are on arithmetic and algebra, due to their hierarchical structure and their importance also for other domains of mathematics (e.g., geometry) which is compatible both nationally and across countries.

The following expand on two examples to clarify the understanding of key skills.

Primary level: executing addition in a structured way
The task 25 + 7 requires more than stepwise counting. A robust operation sense shows when students recognise part–part–whole relations (e.g. 25 and 7 as parts of a whole), decompose numbers flexibly (e.g. 7 = 5 + 2) and build on the next ten (e.g. 25 + 5 = 30; then +2 = 32). Here, concepts (place value, equality), competences (flexible calculation, justified procedure), and the resulting skill (structured addition) work together. If this key skill is missing, the next "level", larger number ranges or more efficient strategies, remains difficult to access.

Lower secondary: managing extension of number domains
A secured operation sense with natural numbers (decomposition, inverse operations, place-value and number-line references) is a prerequisite for transferring procedures to decimals and fractions (e.g. addition/subtraction, rounding, estimation) to overcome epistemological obstacles involved in learning mathematical concepts (Brousseau, 1997). Gaps in these key skills often lead to procedural work without understanding, which in turn impedes access to algebraic expressions, equations and functional relationships. This illustrates the predictive character of arithmetical key skills for algebraic demands.

The key skill understanding is integrated in the tests to

- represent the necessary prerequisites for the next learning step, and

- are content-proximate and thus observable with short tasks, and

- offer teachers a first structured orientation as to which students may require further diagnostic steps and where support can be targeted. The aim is not to assign labels but to reveal central prerequisites early, so that subsequent learning can proceed on a stable basis.

In our understanding, every content domain contains key skills and these may become critical at different points in a learning progression, including at the end of a unit when a capability is needed to enable subsequent learning. The development of key skills is therefore ongoing across grades; identifying missing prerequisites early remains essential so that learners can continue to acquire new content with understanding.

III. Test structure Screening 6+ and 8+

The test structure in DiToM is based on the content areas of arithmetic and algebra. The hierarchical structure of the content area is taken into account. The test construction focussed on the area of number range development and extension in the sense of a technical calculation, insofar as calculation procedures are carried out non-algorithmically and algorithmically on the basis of a fundamental understanding. The diagram shows the test structure in this content area for grades 6+ and 8+.

The test for grade 6+ is based on the building blocks of grade 4+, which focus on natural numbers and this area is more differentiated. If pupils have considerable difficulties in the area of natural numbers in grade 6+, it is recommended to use the test for grade 4+.

|		operations		
			operational under-standing mult. & div. (6)	
	operational understanding addition & subtraction (6)		operational under-standing add. & subtr. (6)	
calculating simple terms (6)	Comparing (6)		Comparing (6)	
operational understanding multiplication & division (6)	(extended) understanding of place-value (6)		reducing / expanding (6)	operational understanding algebraic signs (8)
			identification and realisation (6,8)	
natural numbers	**decimal numbers**		**fractions**	**whole numbers**

In the area of algebra or pre-algebra, the structural understanding of simple mathematical structures in both internal and external mathematical applications is assessed under the aspect of proportionality and linearity. Likewise, in the area of terms with numbers or variables in different directions in basic application situations as well as for term understanding, insofar as it is part of a basic understanding.

proportionality	linearity	terms with variables
basic calculation of percentages (8)		
parts – Identifying percentages (8)		
simple modelling with proportional relationships (6;8)	simple modelling with linear relationships (8)	simple modelling with terms and equations (8)
recognising simple proportional relationships (8)	recognising simple linear relationships (8)	calculating with simple terms and equations (8)
recognising patterns (6)		

4

IV. Implementation of the DiToM test

- Explain the purpose of the test to students and reassure them

- The test is not graded.

- It enables them to take stock of what they know and what they don't know, so that they can then suggest appropriate exercises. Therefore, it is particularly important that they work alone.

- Emphasize the importance of completing the exercises. The more they answer the questions, the easier it will be to identify their knowledge, skills and difficulties, and to help them overcome them.

- You can also say that this is the first time this test has been used and that the people who designed it want to know if it's suitable.

- Test structure

- The test is divided into three parts, each made up of several exercises.

- All exercises are independent of each other.

- Duration: A maximum duration is estimated for each part.

- Test grade 6, maximum 45 minutes: 15 for pre-algebra, 10 for proportionality, 20 for arithmetic.

- Test grade 8, maximum 40 minutes: 15 for pre-algebra, 10 for proportionality, 15 for arithmetic.

- It is important to indicate the duration of each part to students before the test is taken, and that the teacher will interrupt students who have not finished, out of fairness between students.

- Exercise format

- Open-ended exercises: there's space to answer (either with sentences or with a number).

- Closed exercises (multiple-choice questions): several answers are proposed, and the student must respond by choosing just one. Please tell the students that if they decide to change their multiple-choice answer, they should write 'No' beside the first answer and 'Yes' beside the new one.

- How to answer

- Calculators are not permitted.

- Students may use any part of the page left blank as rough draft, in particular to write down their calculations.

- Students can work through the three parts in order, at their own pace. Students who have completed one part of the test should wait for the teacher to give directions to continue with the next part.

- Soliciting students during the test

- If the teacher is solicited, he/she gives no indication that would guide the answer to the questions. The aim is to identify students' difficulties.

# V. Presentation of the tasks

Task 1.1: Perimeter of a rectangle

The perimeter of a polygon is the sum of the lengths of its sides.
Which is the formula for the **perimeter (P)** of the rectangle?

a

4
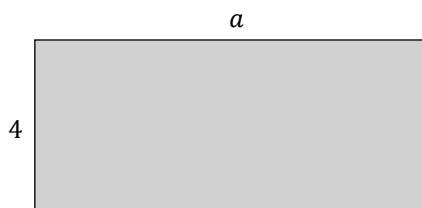
$P = \underline{\text{ 2a + 8 }}$

Solution

The lengths of all four sides must be added.

a

4   4

a
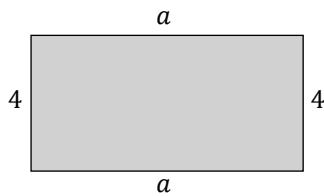
Even if the task asks for "the" formula, several (equivalent) answers are correct.

For example:

$a + 4 + a + 4$ (or $a + a + 4 + 4$, or terms in any order)

$2a + 8$ (or $2(a + 4)$ or $(a + 4) \cdot 2$ or $2 \cdot a + 2 \cdot 4$)

Key skill tested with this task

This task assesses students ability to construct a symbolic formula for the perimeter of a rectangle when given side lengths in algebraic form. The rectangle is labeled with side lengths a and 4 and the task requires students to express the perimeter P as a general expression, rather than computing a numerical value. To solve the task correctly, students must understand that a rectangle has two pairs of sides of equal lengths and that the chosen formula has to add the lengths of all four sides.

$P = 2 \cdot (a + 4)$
$P = (a + 4) \cdot 2$
or equivalently: P=2a + 8   P = 2a + 2 · 4

Or: a+4+a+4 or similar

Why is this skill a key skill?

This task supports the development of algebraic thinking by encouraging students to represent relationships between quantities using variables. Within the DiToM framework, constructing such formulas is essential for building symbolic fluency and understanding functional relationships. It connects geometric reasoning (perimeter) with algebraic expression and reinforces the use of variables as generalized numbers. This skill also prepares students for later work with functions, parameterized models and equation solving.

What kind of errors and other warning signals can be expected with this task?

A very common misconception is that students attempt to calculate a numerical value for the side labeled a—for example, by measuring it directly from the diagram—rather than recognizing that a is a variable and the task requires a general formula, not a specific result. This indicates misalignment between algebraic representation and concrete calculation. Another frequent error is that students compute the perimeter numerically by plugging in estimated or assumed values (e.g., "a is about 6 cm, so..."), showing they misunderstood the nature of the task. This reveals a misinterpretation of the task format—treating it as a computation problem rather than a symbolic generalization. Additionally, some students provide the formula for the area of the rectangle instead of the perimeter, such as writing $A = a \cdot 4$ or $A = 4a$, confusing area and perimeter concepts. This suggests either conceptual confusion between the two properties or difficulties in extracting relevant information from the text.

Altogether, these errors signal weaknesses in:

- Understanding the role of variables

- Distinguishing different geometric concepts

- Precisely interpreting mathematical instructions

What kind of support could be given to children who show deficits with this task?

Students benefit from practice that clearly separates tasks that ask for formulas from those that ask for numerical results. Teachers can use physical cut-out rectangles or digital manipulatives where students label sides with letters and "walk" around the shape to count the total length. Visual supports like algebra tiles or perimeter frames help illustrate why opposite sides are equal and must be summed twice. Encouraging verbal formulations ("two times the sum of a and 4") helps bridge concrete understanding with algebraic notation. Further, contrasting tasks (e.g., "1) Write a formula for the perimeter, in terms of the variable/unknown a; 2) Use the formula to calculate the value of the perimeter for a=5"), can train students to read instructions precisely.

Task 1.2: Translating verbal phrases into algebraic expressions (A–C)

a) The sum of 3 and x $3 + x$

b) 3 less than x $x - 3$

c) Double of a $2a$

Solution

a) The sum of 3 and x can be written as either $3 + x$ or $x + 3$.

b) 3 less than x can be written as $x - 3$ (or $x + (-3)$ or $-3 + x$).

c) Double of a can be written as $2 \cdot a$ or $2a$ or $a + a$.

Key skill tested with this task

This task set assesses students ability to translate everyday language into algebraic notation. Each part (a-d) presents a different verbal structure that corresponds to a simple symbolic expression. In part a (*the sum of 3 and x*), the goal is to recognize additive structure (3 + x). Part b (*3 less than x*) tests whether students correctly reflect a subtractive relationship with the correct operand order (x – 3). Part c (*the double of a*) demands multiplication as scaling (2a). This task combines basic algebraic conventions with linguistic precision, asking learners to navigate between natural language and symbolic expression.

Why is this skill a key skill?

The ability to move fluently between verbal descriptions and algebraic representations is foundational in mathematical literacy. It supports understanding of functional relationships, lays the groundwork for interpreting and writing equations and formulas and is essential for modeling problems. According to the DiToM framework, this ability is central for building symbolic understanding and for developing a meaningful grasp of operations beyond rote procedure. Students who master this translation process are better equipped to interpret word problems and construct equations in more complex settings.

What kind of errors and other warning signals can be expected with this task?

This task reveals characteristic misconceptions at different levels. In part b (*3 less than x*), many students incorrectly reverse the order and write 3 – x, interpreting it as "3 minus x" instead of the intended x – 3. This indicates a difficulty in recognizing relational direction in verbal structures. An error is the confusion between square and double which is often performed.
Collectively, these errors indicate whether students can parse verbal structure, respect algebraic syntax and understand standard mathematical conventions.

What kind of support could be given to children who show deficits with this task?

To strengthen this skill, instruction should emphasize the structure of language in relation to symbolic form. Explicit teaching of phrases like "less than," "more than," "the product of," and "the square of" is essential, ideally with contrast examples. Visual organizers that pair phrases with algebraic forms can aid memorization and structural understanding. Finally, encouraging students to verbalize their expressions ("x minus 3 means I start with x and subtract 3") promotes reflective accuracy and connects language more firmly with mathematical meaning.

Task 1.3: Building an expression from a multistep written description

Laura has 10 books more than Jenny.
Kevin has twice as many books as Laura.

Which is the number of Kevin's books if the number of Jenny's books is n ?

- ☐ $10 + n$
- ☐ $10 + n + 2$
- ☑ $2 \cdot (n + 10)$
- ☐ $2 \cdot n + 10$

Solution

First solution, for a student who is used to working with variables:

Step 1   Jenny has n books

Step 2   Laura has 10 books more than Jenny, so Laura has $n + 10$ books

Step 3   Kevin has twice as many books as Laura, so Kevin has $2 \cdot (n + 10)$ books.

Second solution, based on informal notation:

Laura = Jenny + 10

Kevin = $2 \cdot$ Laura = $2 \cdot (\text{Jenny} + 10) = 2 \cdot (n + 10)$

Third solution, based on testing with a number and then generalizing:

Assume that Jenny has 17 books. Then Laura has $17 + 10 = 27$ books and Kevin has $2 \cdot 27 = 54$ books.

The number 17 is hidden in 27 and the answer $2 \cdot 27 = 2 \cdot (17 + 10)$.

Generalizing by replacing 17 with n gives the answer $2 \cdot (n + 10)$.

Key skill tested with this task

This task assesses students' ability to translate a two-step verbal description into a symbolic expression. The problem describes that Jenny owns x books, Laura owns 10 more than Jenny and Kevin owns twice as many as Laura. To correctly solve the task, students must recognize that Laura's quantity is expressed as n + 10 and then apply multiplication to find Kevin's total, resulting in the expression 2 · (n + 10). This demands attention to both the sequence of relationships and the structural role of grouping in algebraic expressions (this is done with parentheses).

Why is this skill a key skill?

The ability to construct symbolic expressions from relational descriptions is foundational in algebra. It demonstrates a student's capacity to identify functional dependencies and encode them structurally based on the role of parentheses. Within the DiToM framework, such a task represents a key mathematical skill because it requires the coordination of multiple quantities and operations and supports the transition from arithmetic reasoning to algebraic generalization. By working with abstract placeholders and nesting operations, students

begin to think in terms of relationships between variables—an essential step toward modeling, equation solving and functional reasoning.

What kind of errors and other warning signals can be expected with this task?

One common error is that students simplify the structure prematurely by giving the expression 2n + 10 instead of the correct 2 · (n + 10), indicating that they did not taking into account the priority of multiplication over addition. Others may stop after the first relation and simply write n + 10, representing Laura's books rather than Kevin's. A frequent misunderstanding 10+n+2 also lies in interpreting "twice as many" as a separate increase rather than multiplication or in disregarding priority of multiplication with respect addition, which can result in a misinterpretation of the intended calculation. In some cases, students may confuse the characters or the flow of information, indicating that they have difficulty extracting or retaining multiple relational statements from the text. These difficulties point to gaps in structural understanding, especially when combining additive and multiplicative reasoning in symbolic form.

What kind of support could be given to children who show deficits with this task?

Students benefit from explicit verbal unpacking of the relationships, guided by prompting questions like "What does Laura have in terms of Jenny's books?" and "What does Kevin do with Laura's amount?" Using visual aids such as relationship diagrams or boxes helps make these connections more tangible. Additionally, practicing verbal-to-symbolic translation with simpler one-step relations builds confidence before combining steps. Instruction should also explicitly address the use of brackets, contrasting expressions like 2n + 10 and 2 · (n + 10) with concrete numerical examples (e.g. with a spreadsheet) to illustrate the impact of structure. Encouraging students to articulate their reasoning aloud helps reinforce both operational logic and algebraic meaning.

Task 1.4: Building a structured expression from a written description

A group of 13 friends goes to the cinema.

Each of them pays a ticket at x € and buys popcorn for € 3,20.

Which of the following expressions indicates the price paid by the whole group?

- ☐ $13 + (x + 3{,}20)$
- ☐ $x \cdot (13 + 3{,}20)$
- ☐ $13 \cdot x + 3{,}20$
- ☑ $13 \cdot (x + 3{,}20)$

Solution

First solution: Each person pays $x + 3{,}20$ and 13 persons pays 13 times what one person pays.

So the answer is $13 \cdot (x + 3{,}20)$.

Second solution, based on testing with a number and generalizing:

Assume that one ticket costs 5 €.

Each person pays a ticket at 5 € and buys popcorn for € 3,20, so each person pays 8,20 €.

13 persons pay 13 times that amount: $13 \cdot 8{,}20$. This product does not need to be calculated, what is important is what happened to the chosen number 5: $13 \cdot 8{,}20 = 13 \cdot (\mathbf{5} + 3{,}20) = 13 \cdot (x + 3{,}20)$.

The second solution can also be executed by comparison and imitation ("follow the number"):

With number	With variable
$5 + 3{,}20$	$x + 3{,}20$
$13 \cdot (5 + 3{,}20)$	$13 \cdot (x + 3{,}20)$

Key skill tested with this task

This task requires students to translate a written situation into a structured algebraic expression. The context is presented in writing: 13 friends each buy a cinema ticket for x euros and spend an additional €3.20 on popcorn. The total cost for the group must be represented by an expression that combines both components – the variable and the constant – and multiplies the full individual cost by the number of people. The correct expression is therefore 13 · (x + 3.20). To select the right solution, students need to both comprehend the narrative structure and understand how to model repeated addition through multiplication using parentheses.

Why is this skill a key skill?

Representing contextual situations as algebraic expressions reflects a core competence in mathematical modeling. This task exemplifies a fundamental transition from arithmetic reasoning to algebraic structuring: rather than computing or estimating, students are asked to generalize and represent a relationship. According to the DiToM framework, this ability plays a central role in developing functional thinking, structural awareness and a flexible understanding of variable use. It also supports a deep understanding of operations, grouping and the distributive property – skills essential for the manipulation of expressions and equations.

What kind of errors and other warning signals can be expected with this task?

11

Many students tend to overlook the need for brackets and select expressions such as 13x + 3.20. Others might reverse the multiplication, choosing x + 13 · 3.20 or mistakenly represent only a part of the situation. These distractor choices are deliberately designed to reveal specific misconceptions: for instance, failing to apply the distributive structure or misinterpreting how repetition of costs is modeled in algebra. In some cases, students also opt for the numerically simplest-looking option without analyzing the meaning behind it, suggesting superficial reading or reliance on heuristics rather than relational reasoning.

What kind of support could be given to children who show deficits with this task?

To strengthen students understanding, it can be helpful to visualize the scenario using tables or diagrams, showing one row per person and totaling costs column-wise. Explicit modeling of similar tasks – such as "each person pays €x and €y, what's the total for n people?" – builds familiarity with grouping structures. Emphasizing bracket use through spoken phrases ("the whole thing per person, then times 13") supports students in transferring verbal structures into algebraic form. Additionally, discussions comparing the given options and evaluating what each expression represents can help students build metacognitive strategies for interpreting symbolic choices. Also Encourage the student (who does not know how to begin) to begin by testing with any (self-chosen) number.

Task 1.5: Structuring a written expression from a multistep rule

A calculation is defined in steps:

  – Choose a number x

  – Add 4 to x

  – Multiply the result by 8

Which of the following expressions indicates the calculation?

☐ $8 \cdot x + 4$
☐ $x + 4 \cdot 8$
☑ $(x + 4) \cdot 8$
☐ $(8 \cdot 4) + x$

Solution

First solution, in two steps.

Step 1 Adding 4 to x gives the result $x + 4$

Step 2 Multiplying the result $x + 4$ by 8 gives the answer $8 \cdot (x + 4) = (x + 4) \cdot 8$.

Second solution, based on testing with a number and generalizing: Assume that $x = 17$.

Step 1 Adding 4 to 17 gives the result $17 + 4 = 21$

Step 2 Multiplying the result 21 by 8 gives the answer $21 \cdot 8$.

Step 3 Analyze what happened to 17: $21 \cdot 8 = (17 + 4) \cdot 8 = (x + 4) \cdot 8$.

As in the solution to the previous task, we can also "follow the number" to find the answer:

With number	With variable
$17 + 4$	$x + 4$
$8 \cdot (17 + 4)$	$8 \cdot (x + 4)$

Key skill tested with this task

This task asks students to represent a sequential calculation—specifically, to symbolically express the outcome of adding 4 to x and then multiplying the result by 8. Although the verbal instructions mention additional steps, the answer choices only refer to this first operation chain. The correct expression in this reduced context is (x + 4) · 8, which requires students to preserve the order of operations by grouping x + 4 before applying multiplication.

Why is this skill a key skill?

Understanding and representing the hierarchical structure of operations is a fundamental skill in algebra. It allows students to move from interpreting arithmetic step-by-step to organizing these steps in structured symbolic form. According to the DiToM framework, this is part of developing symbolic modeling competencies, especially the

use of parentheses to clarify operation precedence. This is not only crucial for evaluating expressions correctly, but also for building confidence in manipulating formulas and solving equations later on.

What kind of errors and other warning signals can be expected with this task?

The distractors are carefully chosen to reveal specific structural misunderstandings. For example, $8 \cdot x + 4$ represents the error of performing the multiplication too early and then incorrectly adding 4, violating the intended grouping. The choice $x + 4 \cdot 8$ shows linear reading and mistaken precedence, where students apply multiplication before addition without bracketing. $8 \cdot 4 + x$ ignores the structure entirely, suggesting a superficial focus on the numbers involved rather than the relationship. These answer choices are not random—they signal whether a student understands how brackets affect operation order and whether they can parse multi-step verbal instructions structurally rather than sequentially.

What kind of support could be given to children who show deficits with this task?

Support should aim to strengthen students bracket awareness and their understanding of operation precedence. A helpful approach is to let students verbalize each step, then physically group expressions using cards or color-coding. They should be encouraged to compare expressions like $x + 4 \cdot 8$ and $(x + 4) \cdot 8$ with numerical substitutions (e.g., $x = 2$) to test whether the structure matches the intended meaning. Explaining why brackets are necessary in this context supports symbolic reasoning and helps move students beyond procedural translation toward genuine understanding. Also students can begin by testing self-chosen numbers and generalize their calculation.

Task 1.6: Evaluating an expression with variable substitution

What is the value of $1 + 3x$ for $x = 8$?

- ☑ 25
- ☐ 32
- ☐ 39
- ☐ 48

Solution

The key to solving this problem is noting that $3x = 3 \cdot x$ has to be calculated before the number 1 is added.

Solution in one step: Set $x = 8$ and calculate

$$1 + 3x = 1 + 3 \cdot 8 = 1 + 24 = 25$$

Solution in two steps: Set $x = 8$. First calculate $3x = 3 \cdot 8 = 24$. Then calculate $1 + 24 = 25$.

To get a better overview of the calculations in two steps, we can present them on separate rows:

$$x = 8$$

$$3x = 3 \cdot x = 3 \cdot 8 = 24$$

$$1 + 3x = 1 + 24 = 25$$

Key skill tested with this task

This task assesses students ability to evaluate an algebraic expression by substituting a given value for the variable and applying the correct order of operations. Specifically, students are given the expression 1 + 3x and must compute its value for x = 8. Recognizing that 3x means 3 times x, the correct procedure is to multiply first:

1+3·8=1+24=25

Students need to correctly substitute the variable and then execute the operations in the proper sequence, respecting the priority of operations.

Why is this skill a key skill?

Substitution is one of the most fundamental processes in algebra and forms a bridge between symbolic expressions and numerical reasoning. Within the DiToM framework, evaluating expressions by substituting values helps develop operational fluency, symbol sense and confidence with variables. These skills are foundational for engaging with functions, constructing tables and graphs and solving real-world problems algebraically. It also promotes a flexible understanding of mathematical structure, especially the relationship between symbols, operations and their meaning.

What kind of errors and other warning signals can be expected with this task?

Common errors involve misunderstanding the expression structure or misapplying the order of operations. Some students may compute 1 + (3 + 8) = 12, misreading 3x as 3 + x rather than 3 times x. Others might incorrectly evaluate 3x as 38 (concatenating 3 and 8), which reflects symbolic misinterpretation rather than calculation error. Students who choose 32 may have added 1 to 8 first and then multiplied the sum by 3: (1 + 8) · 3 = 27 or misread the term as (1 + 3) · 8 = 32, introducing unintended brackets. Selecting 39 or 48 may suggest incorrect doubling,

arbitrary estimation or total disregard of order. These distractors reveal whether students understand how variables operate within expressions and how order of operations functions without explicit bracketing.

What kind of support could be given to children who show deficits with this task?

Instruction should emphasize reading algebraic expressions structurally and not procedurally. Teachers can support learning by encouraging students to verbalize expressions ("one plus three times x") and connect these to numerical evaluation. Using substitution tables that clearly separate each operation helps clarify structure. Comparison activities, such as evaluating both 1 + 3x and (1 + 3)x, help highlight how brackets change meaning, while dynamic tools can allow students to test various values interactively. Reinforcement of order of operations through structured practice, especially in contexts without parentheses, supports long-term accuracy and fluency.

Task 1.7: Identifying the solution to a linear equation

The equality $7x + 3 = 80$ is verified for

☐ $x = 7$

☐ $x = 8$

☐ $x = 10$

☑ $x = 11$

Solution

First solution, based on testing all the four numbers:

x	7	8	10	11
$7x + 3$	$49 + 3 = 52$	$56 + 3 = 59$	$70 + 3 = 73$	$77 + 3 = 80$

Matches the given number 80 in the equation

Second solution, based on calculating $7x + 3$ with some of the suggested numbers: We can start with any of the four numbers, for example $x = 10$ that gives the simple calculation

$$7x + 3 = 7 \cdot 10 + 3 = 70 + 3 = 73 \quad \text{(does not match 80)}$$

The answer 73 is less than 80 so we need a number larger than 10. The only such number among the four is $x = 11$ that gives the calculation

$$7x + 3 = 7 \cdot 11 + 3 = 77 + 3 = 80 \quad \text{(matches the given number 80 in the equation)}$$

Hence, we can conclude that the correct answer is $x = 11$.

Third solution, solving symbolically (short version):

$7x + 3 = 80$ (subtract 3)

$7x = 77$ (divide by 7)

$$x = \frac{77}{7} = 11$$

Key skill tested with this task

This task evaluates whether students understand the concept of a linear equation and can determine whether a given number is a solution by substituting values. Students are presented with the equation 7x + 3 = 80 and asked to identify the value of x that makes this statement true. The correct response is x = 11, since substituting this value yields a true equation:

7·11+3=80

To answer correctly, students must not only perform the calculation but also understand that a solution to an equation is a number that balances both sides.

Why is this skill a key skill?

Understanding equations as statements of equality and knowing how to verify them is fundamental to algebra. In the DiToM framework, this competency belongs to the area of symbolic interpretation and equality reasoning.

It prepares students to solve equations systematically and to check whether a candidate solution satisfies an equation. This understanding supports higher-level algebraic thinking and strengthens number sense through inverse reasoning and operational control.

What kind of errors and other warning signals can be expected with this task?

Some students may substitute the values mechanically but make calculation errors in evaluating $7x + 3$. Others may misunderstand what the equation asks, interpreting the expression as a calculation task rather than a conditional statement. A common misconception is selecting $x = 10$, simply because it "looks close," which suggests estimation rather than reasoning. Some might choose $x = 8$ or $x = 7$ based on guessing or incomplete substitution, possibly stopping once the left side seems near 80. Or replacing x by 7 as a digit of a number in the decimal system. These errors reveal gaps in understanding the meaning of equality, as well as weakness in substitution procedures and arithmetic accuracy.

What kind of support could be given to children who show deficits with this task?

Students should be guided through activities that emphasize what it means for a value to satisfy an equation. Structured substitution tables where students check multiple values for a single equation help build intuition. Teachers can also encourage verbal reflection ("Does the left side equal the right side?") and promote estimation strategies alongside exact calculations. Using balance-scale metaphors or manipulatives supports a conceptual grasp of equality. Finally, giving students opportunities to create their own equations and test values helps foster ownership and deeper understanding of variable relationships.

Task 2.1: Solving a proportional reasoning problem in a real-world context

2 kg of potatoes cost 2,40 €. Determine the price of 5 kg.

6 €

Solution

Solution based on (creative) scaling: 2 kg costs 2,40 €, so 4 kg costs 4,80 €. An additional 1 kg costs 1,20 €, so 5 kg costs $4{,}80 + 1{,}20 = 6$ €.

Second solution, based on scaling down to one unit: 2 kg costs 2,40 €, so 1 kg costs 1,20 €. 5 kg costs 5 times as much: $5 \cdot 1{,}20 = 6$.

Both solutions can be supported by pictures. For example, regarding the last solution:
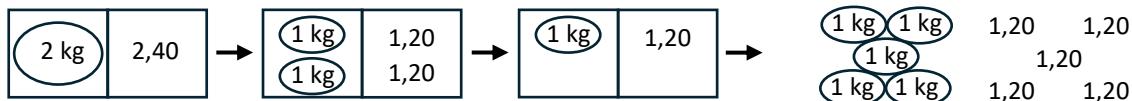
With semi-formal symbolic notation:

2 kg : 2,40 €   (divide by 2)
1 kg : 1,20 €   (multiply by 5)
5 kg : 6 €

Key skill tested with this task

This task requires students to apply multiplicative reasoning to solve a proportionality problem embedded in a familiar everyday context. Given that 2 kg of potatoes cost €2.40, students must determine the price for 5 kg. This implies recognizing a constant rate per kilogram (€1.20 per kg) and scaling this to a new quantity. The correct approach involves either unit rate reasoning (dividing €2.40 by 2 and multiplying by 5) or constructing and solving a proportion. Or interpreting 5=2+2+1 and adding 2.40+2.40+1.20. The task tests not just procedural ability, but also the conceptual understanding of how quantities grow in direct proportion.

Why is this skill a key skill?

Proportional reasoning is a cornerstone of mathematical thinking. It enables students to interpret and model real-world relationships involving scaling, unit rates and multiplicative comparisons. Within the DiToM framework, this type of task supports key developmental steps toward functional thinking and prepares learners for advanced topics such as linear functions and percentage calculations. Additionally, it promotes number sense and flexibility in applying different strategies to proportional situations.

What kind of errors and other warning signals can be expected with this task?

Students might incorrectly use additive reasoning, such as calculating "2 kg → €2.40, so 5 kg → €2.40 + €5" or other inconsistent jumps, indicating they do not grasp the proportional structure. Others might attempt to scale by 2 and then subtract (e.g., 2 kg → €2.40, 4 kg → €4.80, then somehow derive 5 kg from there), which may reflect partial strategy use without aquiring necessary prerequisites. Some learners simply guess a price that "seems reasonable," which shows surface-level context familiarity but weak quantitative reasoning. Errors may

also arise from inaccurate computation or dividing the wrong way around (e.g., dividing 5 by 2.40 instead of the inverse). These patterns indicate gaps in scaling logic, unit rate understanding or interpretation of real-life quantities.

What kind of support could be given to children who show deficits with this task?

Support should center on building strong intuitive and procedural understanding of proportionality. Using double number lines or ratio tables allows students to visualize relationships and scale up incrementally. Teachers can pose verbal scaffold questions like "How much does 1 kg cost?" or "What happens if you buy twice as much?" Encouraging multiple strategies—unit rate, proportion setup, scaling in steps—helps students develop flexibility and error-checking habits. Linking such problems to real shopping experiences may also strengthen engagement and sense-making.

Task 2.2: Solving a problem involving inverse proportionality

If using four equal taps, a pool has taken 6 hours to fill, how many equal taps will it take to fill a pool with the same capacity in 2 hours?
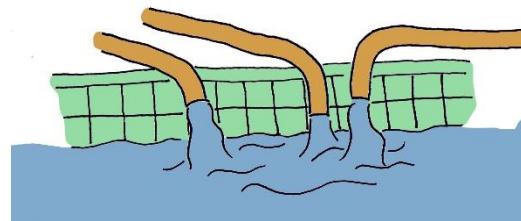
Solution

Solution based on proportional reasoning: Since four taps fill the pool in 6 hours, each tap fills $\frac{1}{4}$ pool in 6 hours, and $\frac{1}{12}$ pool in 2 hours. We conclude that 12 taps are needed to fill the pool in 2 hours.

To support reasoning step by step, this solution can be presented in semi-symbolic notation:

4 taps fill 1 pool in 6 hours

1 tap fills $\frac{1}{4}$ pool in 6 hours

1 tap fills $\frac{1}{12}$ pool in 2 hours

12 tap fills 1 pool in 2 hours

Similar reasoning, based on a formula: The volume of the added water is proportional to (#taps) · (time). We know that (ignoring units)

$$(4 \text{ taps}) \cdot (6 \text{ hours}) = 1 \text{ pool}$$

$$(1 \text{ taps}) \cdot (6 \text{ hours}) = \tfrac{1}{4} \text{ pool}$$

$$(1 \text{ taps}) \cdot (2 \text{ hours}) = \tfrac{1}{12} \text{ pool}$$

$$(12 \text{ taps}) \cdot (2 \text{ hours}) = 1 \text{ pool}$$

Key skill tested with this task

This task involves a real-world scenario where students must understand the relationship between time and quantity in a filling process. Specifically, four identical taps fill a pool in 6 hours. The question now is: how many taps are needed to fill the same pool in only 2 hours? To solve the task, students must recognize that the faster the task needs to be completed, the more taps are required. While not explicitly stated, this describes a case of inverse proportionality, where one quantity increases as the other decreases. A valid and accessible strategy involves organizing the known and desired values in a table or ratio diagram:

Number of taps	Time (in hours)
4	6

Number of taps	Time (in hours)
?	2

From here, students can reason that since the time is divided by 3 (from 6 to 2 hours), the number of taps must be multiplied by 3, leading to: 4·3=12 taps

This strategy emphasizes relational thinking and supports a proportional reasoning approach without invoking formulaic or abstract methods.

Why is this skill a key skill?

The ability to understand relationships where quantities interact in opposing directions is a key aspect of flexible multiplicative thinking. In contrast to direct proportions, inverse relationships demand a different kind of structural awareness. According to the DiToM framework, mastering this helps students extend their quantitative reasoning and prepares them for advanced topics such as work problems, rates and functional thinking. Situations like this also cultivate students skills in modeling and making sense of real-world relationships using mathematical tools.

What kind of errors and other warning signals can be expected with this task?

Some students may misinterpret the scenario as a case of direct proportion, assuming that reducing the time should also reduce the number of taps—potentially arriving at incorrect answers like 1 tap or 2 taps. Others may apply arithmetic strategies inconsistently or reverse the proportionality, dividing the number of taps instead of increasing them. A frequent misconception is to treat the situation additively rather than multiplicatively (e.g., "2 is 4 less than 6, so subtract 4 taps"). These patterns signal a lack of familiarity with inverse structures and the role of scaling in opposite directions.

What kind of support could be given to children who show deficits with this task?

Instruction can begin with visual models or concrete examples that show how more people or devices working together complete a task faster. Using double number lines or structured tables like the one above helps make the multiplicative relationships explicit. It is also useful to pose guided questions, such as "What happens if you need to finish in half the time?" or "How does the time change when you double the number of taps?" Providing paired comparison tasks—one direct, one inverse—can help highlight structural differences. Encouraging students to explain their reasoning aloud or compare different solution paths also strengthens understanding.

Task 2.3: Identifying proportional relationships in tabular representations

Cross if the price we pay is directly proportional to the number of cakes we buy?

a. Table 1

Cakes	1	2	5
Price	5	10	50

☐

b. Table 2

Cakes	1	2	5
Price	11	12	15

☐

c. Table 3

Cakes	1	2	5
Price	3	6	15

☒

Solution

Two common strategies for identifying direct proportionality between two variables:

1) When any two proportions are compared (for example, in Table 1: 1:5 and 2:10) the variables should change by the same factor (in this case 2). However, when comparing 1:5 and 5:50, the variables change by different factors: 5 and 10. Hence, Table 1 does not represent a proportional relationship.

2) In each proportion, the quotient of the variables should be the same (for example $\frac{1}{5} = \frac{2}{10}$ or $\frac{5}{1} = \frac{10}{2}$). In this task, the quotient $\frac{price}{number\ of\ cookies}$ can be interpreted as the price per cookie. In Table 1, the price per cookie is $5/1 = 5$, $10/2 = 5$, $50/5 = 10$, depending on how many cookies we buy. Hence, Table 1 does not represent a proportional relationship.

Table 1, strategy 1 represented in a table:

Factor		2	2,5
Cakes	1	2	5
Price	5	10	50
Factor		2	5

Not OK

Table 1, strategy 2 presented in a table:

Cakes	1	2	5
Price	5	10	50
Quotient	5	5	10

Not OK (different prices 5, 5, 10 per cookie)

In Table 2, it can be readily observed that the proportions 11:1 and 12:2 are not equal. Hence, no further work is needed to reach the conclusion that the variables are not proportional.

In Table 3, the unit price is 3 for each of the proportions and hence the variables are directly proportional.

Key skill tested with this task

This task assesses students ability to analyze tabular data to determine whether a proportional relationship exists between two quantities. In each of the three tables, learners are expected to examine whether the relationship between the columns can be described by a constant multiplicative factor, which is the defining feature of direct proportionality. The students must cross if the table is direct proportional.

To solve this task successfully, a few examples for solving:

- Check whether the quotients (e.g., y : x) are constant across all rows

- Or check whether the cross-multiplication gives equivalent products

- Or identify if a consistent rule like "multiply by 3" or "double it" applies

This form of analysis requires attention to numerical structure and underlying multiplicative patterns, not just surface features.

Why is this skill a key skill?

Understanding proportional relationships is a fundamental part of functional and algebraic thinking. Within the DiToM framework, this task supports the development of relational reasoning, especially the recognition of structure in data. The ability to determine whether a relationship is proportional builds essential groundwork for interpreting linear functions, scaling problems and graphical representations. It also promotes flexibility in working across different representations—tables, graphs, verbal descriptions and equations.

What kind of errors and other warning signals can be expected with this task?

Some students may treat relationships as proportional based on superficial increases (e.g., "both numbers get bigger") without verifying the constancy of the ratio. Others may compare absolute differences instead of ratios, confusing additive with multiplicative reasoning. For example, if both values increase by 2, students might wrongly conclude proportionality. Errors may also arise from careless computation (e.g., dividing incorrectly or skipping values). These patterns reflect underdeveloped skills in multiplicative comparison, rate interpretation and symbolic generalization. In some cases, students might misclassify a non-proportional table due to the presence of simple-looking numbers or cross all tables to avoid analysis. Such tendencies highlight a need for stronger structuring strategies and confidence in verification.

What kind of support could be given to children who show deficits with this task?

Effective support includes providing students with tools to test proportionality systematically, such as computing ratios row by row, using unit rate analysis or visual aids like double number lines. Teachers can model how to explain findings verbally ("Because 6 : 2 = 3 and 9 : 3 = 3, the ratio is constant") and encourage students to use checklists when analyzing tables. Pairwise discussion tasks—where students defend whether a table is proportional—can deepen reasoning. Linking tables to real-world proportional scenarios (e.g., recipes, prices, speeds) helps ground understanding and foster transfer.

Task 2.4: Working with graphs in a proportional context

The graph shows how the cost (in euro) depends on the volume of water you consume (in m³).
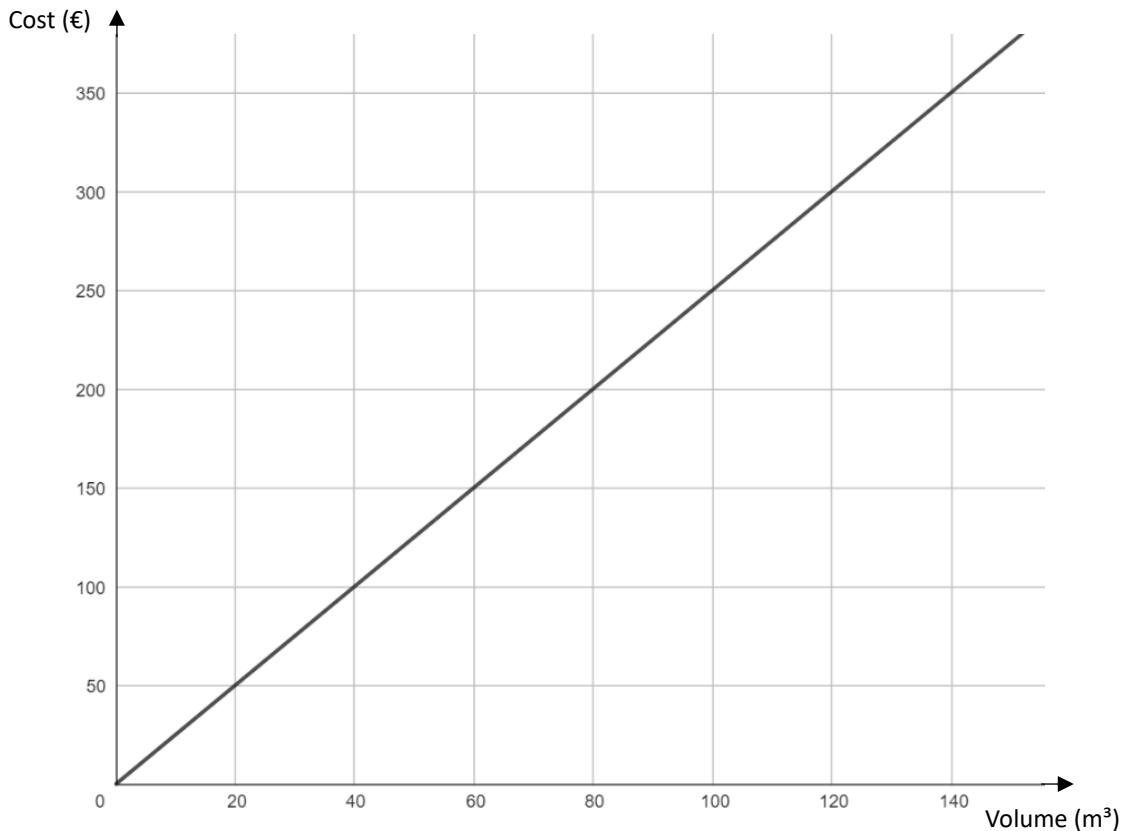
a) Determine how much m³ of water you get for 200 €.

Answer: ___*80 m³*___

b) Determine the cost for 300 m³ water.

Answer: ___*750 €*___

Solution

a)
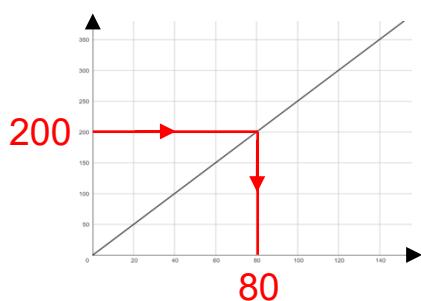
b) Reading values in the graph and scaling
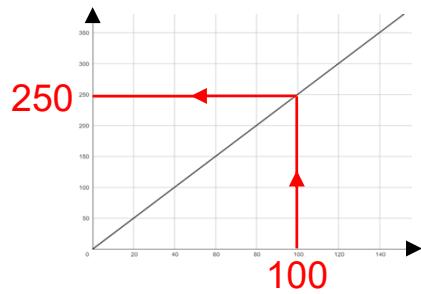
In part b) we begin by observing that 100 m³ costs 250 euros.

We continue arguing that 300 m³ costs 3 times as much as 100 m³, and conclude that 300 m³ costs

$$3 \cdot 250 = 750 \text{ euro}$$

Key skill tested with this task

This two-part task assesses students ability to interpret and apply a linear graph representing a proportional relationship. In Part A, the task is to read off a value directly from the graph: determining how many cubic meters of water are available for €200. In Part B, the student must determine the price for 300 m³ of water – a value beyond the visible range of the graph – and thus must calculate based on the rate of change observed in the graph.

The key skill lies in connecting the graphical representation with underlying proportional reasoning and distinguishing between graph reading and model-based extrapolation.

Why is this skill a key skill?

Reading and interpreting graphs is a core mathematical competency. This task bridges between visual data interpretation and functional thinking. According to the DiToM framework, this type of activity develops the ability to: Understand slope as a rate, Use a graph as a tool for decision-making, Translate between graphical, numerical and verbal representations. Moreover, this skill supports competencies in mathematical modeling, especially in recognizing when data needs to be extended through proportional reasoning rather than estimation.

What kind of errors and other warning signals can be expected with this task?

In Part A, students might misread the axes (e.g., misalign values due to unfamiliar scaling) or interpolate incorrectly between points. If they misunderstand that the graph represents a continuous relationship, they may only reference labeled grid values and skip over intermediary points. In Part B, some students may attempt to extrapolate visually despite the graph ending at 140 m³, leading to imprecise or speculative answers. Others may fail to recognize the linear structure and revert to guessing or unrelated strategies. Another typical error is misinterpreting the slope—for example, thinking it increases nonlinearly or changes unpredictably beyond the visible range. These difficulties may reflect gaps in understanding proportional growth, linear extrapolation or graph-reading conventions (such as equal scaling and alignment of grid lines).

What kind of support could be given to children who show deficits with this task?

Targeted support should begin with scaffolded graph-reading exercises that guide students to align values correctly on both axes, recognize constant rates and use rulers or tracing to maintain orientation. For extrapolation (Part B), instruction should emphasize how to determine the unit rate from the graph (e.g., €1 per m³ or €5 per 20 m³) and then extend that relationship numerically or via a table. Teachers might also encourage students to express in words "what the graph says" (e.g., "for every 20 cubic meters, the cost increases by €50"), reinforcing verbal-numerical-visual connections. Comparing the graph with the corresponding equation or ratio table also strengthens flexible access to the underlying structure.

Task 2.5: Solving a division problem in a real-world context

There are 810 liters of water in a storage tank.

Every day, 30 liters of water are taken from the tank.

Calculate after how many days the tank will be empty.

27 days

Solution

The daily removed 30 liters results in $30 \cdot n$ liters removed after n days.

First solution, based on division: The number of days needed can be expressed as

$$\frac{810}{30} = \frac{81}{3} = 27$$

Second solution, based on multiplication. Here represented in a table:

# days	1	10	20	7	27
volume	30	300	600	210	810

Key skill tested with this task

This task assesses the ability to solve a division problem in a time-based setting, where a total quantity (810 liters of water) is depleted at a constant daily rate (30 liters per day). The question is how many equal daily withdrawals are needed until the storage tank is completely empty. This involves interpreting division as "how many times does 30 fit into 810" or equivalently:

810 : 30=27 days

This interpretation requires an understanding of repeated subtraction (or addition) over time and the ability to translate a real-world process into a symbolic operation.

Why is this skill a key skill?

Within the DiToM framework, this task supports key competencies in multiplicative reasoning, functional thinking and basic algebraic structuring. Students must identify the underlying regularity ("30 liters per day") and extend it across a repeated process. This also reinforces understanding of rates, time modeling and the interpretation of division in non-spatial, process-oriented contexts. Such reasoning underpins later learning in linear functions, growth patterns and difference equations.

What kind of errors and other warning signals can be expected with this task?

In practice, many students approach the task with the following strategies:

- Repeated subtraction: subtracting 30 from 810 again and again and counting the steps

- Repeated addition: adding 30 each time until reaching 810, then counting the number of steps

- Estimation errors: jumping to plausible-sounding numbers like 30 or 25

- Arithmetic slips in performing long division or misplacing decimal points

What kind of support could be given to children who show deficits with this task?

Support should focus on helping students build an internal model of equal partitioning across time. Visual scaffolds such as bars divided into equal parts, number lines or calendars can aid comprehension. Teachers can encourage students to represent the situation with division sentences and to explain what each number represents. Connecting this problem to real-life routines (e.g., daily water use, meal portions, budgeting over days) can enhance familiarity. Emphasizing strategic grouping, such as chunking by 10 days ($10 \cdot 30 = 300$), promotes efficient reasoning and estimation control.

Task 2.6: Combining fixed and variable costs in a production context

Sven invited 15 friends to his birthday party.

He has to pay 50 euro for the playroom and an additional 10 euro for each invited friend.

How much will Sven have to pay for his birthday celebration?

200 euro

Solution

Sven has to pay 50 euro for the playroom, as a starting cost.

In addition, Sven has to pay 10 euros for each of his 15 friends. That results in an extra cost $15 \cdot 10 = 150$ euro.

The total cost is $50 + 150 = 200$ euro.

Key skill tested with this task

This task assesses students ability to apply basic algebraic modeling and arithmetic operations in a real-world context involving fixed and variable costs. Students are told that Sven spends 50€ for a playroom and additonal 10€ for each invited friend. The task is to determine the total cost for the birthday celebration.

The expected strategy is to:

1. Calculate the variable cost:
   15 chains · 10€ = 150€

2. Add the fixed cost:
   150€ + 50€ = 200€ total cost

Why is this skill a key skill?

This task builds important foundations for functional thinking and early algebra, particularly the concept of linear models. Within the DiToM framework, it supports the development of mathematical modeling skills, operational fluency and the interpretation of symbolic expressions in applied situations. Understanding the distinction between fixed and variable components is central to many real-life contexts, such as budgeting, pricing and resource planning. Moreover, this task strengthens students ability to structure multi-step problems, a key feature of intermediate mathematical problem-solving.

What kind of errors and other warning signals can be expected with this task?

A common mistake is for students to only compute the variable cost and omit the fixed cost. This reveals a partial understanding of the task structure, where only one component (often the most immediately calculable one) is considered. Other students may reverse steps (e.g., subtract 50€ from 150€) or misunderstand the role of each number. Some may mistakenly multiply all values together or attempt unnecessary operations, which reflects weak interpretation of the problem setup. These errors signal deficits in reading multi-step tasks, mapping real-world quantities to operations and integrating different numerical components into a single coherent solution.

What kind of support could be given to children who show deficits with this task?

Teachers can support learners by structuring the problem visually—for example, using a table with two rows (fixed and variable) or diagramming the total cost as a bar split into two parts. Explicitly teaching and practicing with the form "total = fixed + variable" in different contexts can build familiarity with linear models.

Additionally, prompting students to re-read the question and verify whether their answer includes "all" costs fosters metacognitive checking. Using word problems that isolate fixed vs. variable costs before combining them can scaffold understanding step-by-step. Finally, verbalising the situation (e.g., "He has to pay 50€ no matter what and 10€ more for each guest") links language and mathematical structure more clearly.

Task 3.1: Converting a fraction to a percentage

Express the fraction $\frac{3}{5}$ as a percentage.

☐ 0,6%
☐ 6%
☐ 35%
☑ 60%

Solution

First solution, by scaling up a number fact:

$$\frac{1}{5} = 0,20 = 20\%, \text{ so } \frac{3}{5} = 3 \cdot 20\% = 60\%$$

Second solution, by scaling the fraction:

$$\frac{3}{5} = \frac{3 \cdot 20}{5 \cdot 20} = \frac{60}{100} = 60\%$$

Third solution, based on division:

$$\frac{3}{5} = \frac{3,0}{5} = 0,6 = 0,60 = 60\%$$

Key skill tested with this task

This task assesses students ability to convert a familiar fraction (three-fifths) into a percentage. To solve it, students must understand the relationship between fractions and percentages and recognize that:

3⁄5 = 0.6 = 60%

3/5=60/100=60%

This involves either converting the fraction into a decimal first (by dividing numerator by denominator) and then multiplying by 100 or applying known fraction-percentage equivalences. The task thus engages both calculation skills and conceptual understanding of proportional representations.

Why is this skill a key skill?

The conversion between fractions, decimals and percentages is fundamental in many real-life contexts (e.g., interpreting statistics, discounts or data comparisons). Within the DiToM framework, this competency supports rational number understanding, number flexibility and symbolic translation. It prepares students for tasks involving percentage growth, comparisons and proportional reasoning in algebra and data literacy. Mastery in this area also builds confidence in navigating between different representations of parts and wholes, which is essential across mathematics domains.

What kind of errors and other warning signals can be expected with this task?

Many students mistakenly treat the fraction as a decimal or confuse the roles of numerator and denominator. For instance:

- Selecting 0.6% suggests that students misinterpret 0.6 as already being a percent

- Choosing 6% likely stems from moving the decimal point incorrectly once

- 35% may result from confusing three-fifth

These errors indicate confusion between relative values, decimal place movement and the meaning of percent ("per hundred"). They reveal a need for stronger understanding of how fractions map onto the hundred-based scale of percentages.

What kind of support could be given to children who show deficits with this task?

Teachers can support learners by using visual models (e.g., 100-grids, pie charts) to show how fractions like 3/5 correspond to 60 shaded squares out of 100. Structured activities that repeatedly convert common benchmark fractions to percentages (like 1/2, 1/4, 3/4, 3/5) can build familiarity and anchor understanding.

Encouraging students to always convert to a decimal first, then multiply by 100, provides a reliable routine. Number lines and percentage bars can further reinforce the idea of equivalent representations. Verbal explanations ("three out of five equals how many out of one hundred?") help solidify proportional thinking.

Task 3.2: Calculating a percentage increase

If 30 is increased by 50% the result is:

- ☐ 80
- ☑ 45
- ☐ 35
- ☐ 15

Solution

First solution, based on interpreting 50% as "half": The increase is half of 30, which is 15. The result after increasing 30 by 15 is $30 + 15 = 45$.

More formally, with division:

$$50\% \text{ of } 30 = \frac{30}{2} = 15 \qquad \text{Total: } 30 + 15 = 45.$$

Second solution, based on calculating 50% increase by multiplication: The increase is

$$50\% \text{ of } 30 = 0{,}50 \cdot 30 = 5 \cdot 3 = 15 \qquad \text{Total: } 30 + 15 = 45.$$

Third solution, based on interpreting the result as 150% of 30:

$$150\% \text{ of } 30 = 1{,}50 \cdot 30 = 15 \cdot 3 = 45$$

Key skill tested with this task

This task assesses whether students are able to correctly interpret and apply a percentage increase to a given base value. The starting value is 30 and the instruction is to increase this value by 50%. To arrive at the correct solution, students need to determine how much 50% of 30 is and then add this to the original amount.

Why is this skill a key skill?

The ability to calculate and interpret percentage increases is a core competence in both everyday life and advanced mathematical contexts. It is essential in understanding changes in prices, population growth, financial interest and statistical comparisons. According to the DiToM framework, percentage-related reasoning supports the development of functional thinking, proportional reasoning and the integration of multiplicative structures across contexts. This task in particular strengthens students flexibility in switching between additive and multiplicative interpretations of percentage and contributes to their ability to make generalizations and apply structure-based reasoning in increasingly abstract problems.

What kind of errors and other warning signals can be expected with this task?

A common misconception that can be observed in students responses is the confusion between absolute values and relative percentage values. Some students may erroneously add 50 to 30 rather than 50% of 30, arriving at an incorrect answer of 80. This indicates a misunderstanding of the fundamental nature of percentage as a relative measure. Another group of students might select 15 as their answer, misinterpreting the task as merely asking for 50% of 30, rather than calculating the full increased value. Others might choose 35 due to minor computational errors or faulty estimation during mental arithmetic. It is also possible that students misread or misinterpret the wording of the problem, assuming it to be a subtraction or difference task or they may rely on

intuition rather than structured computation. Such responses reveal gaps in conceptual understanding, procedural reliability and reading comprehension. They may also signal an insufficient internalization of percentage operations as scalable multiplicative processes and a need for more guided practice in distinguishing between part, whole and growth rate.

What kind of support could be given to children who show deficits with this task?

Teachers can provide scaffolding by modeling real-world situations where percentage increases naturally occur—such as price markups, pay raises or population growth—thereby grounding abstract concepts in tangible examples. Visual aids like percentage bars or double number lines can help students visualize how an increase of 50% expands the original quantity.

Explicit instruction on distinguishing base, percentage and result within a structured framework can prevent confusion and support a more robust understanding. Additionally, using both additive (e.g., "find 50% and then add") and multiplicative (e.g., "multiply by 1.5") strategies and encouraging students to cross-verify results with estimation, helps promote strategic flexibility.

Task 3.3: Interpreting circle representations and converting to percentages

A part of the circle is coloured. Give the part in percentage.

a)
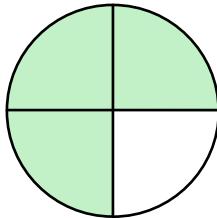
Part of the circle: ___*75*___ %

b)
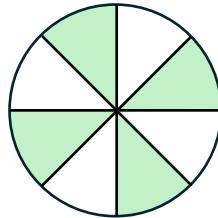
Part of the circle: ___*50*___ %

Solution

a) Each of the four parts are 25% of the circle.
So 3 parts are $3 \cdot 25\% = 75\%$ of the circle.
This reasoning can be supported by drawing a picture.
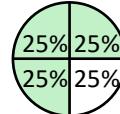
b) Each of the eight parts are 12,5% of the circle (half of 25%).
So 4 parts are $4 \cdot 12,5\% = 50\%$ of the circle.

Another way to solve the problem is to interpret the coloured part as consisting of 4 out of 8 "wedges". Hence, the coloured part is 4 wedges and the total is 8 wedges which either leads directly to the interpretation "half" = 50%, or the calculation

$$\frac{4}{8} = \frac{4/4}{8/4} = \frac{1}{2} = 50\% \, , \qquad \text{(or, similarly, } \frac{4}{8} = \frac{4 \cdot 1}{4 \cdot 2} = \frac{1}{2} = 50\%)$$

The answer 50% can also be obtained by rearranging the four wedges so that the "half" appears more clearly.

Key skill tested with this task

This task assesses students ability to interpret parts of a whole from a visual representation—in this case, a circle divided into equal sectors—and to convert these parts into percentages. The task appears in two parts:

- Part A shows a circle divided into four equal sectors, with three of them shaded.

- Part B presents a circle divided into eight equal sectors, where every second sector is shaded, resulting in four shaded sectors out of eight.

In both cases, the correct solution requires recognizing the fraction of the circle that is shaded and converting it to a percentage. This task involves translating visual quantities into rational numbers or fractions and then into percentage form, demonstrating fluency across representations.

Why is this skill a key skill?

Converting between visual models, fractions and percentages is a central skill in mathematics education. It supports a deeper understanding of part-whole relationships, proportional reasoning and estimation. According to the DiToM framework, tasks like this reinforce conceptual flexibility and the ability to connect mathematical representations—a critical feature of mathematical literacy. The skill is also highly relevant in real-life contexts, such as interpreting charts, statistics and data visualizations, where part-whole relationships are often embedded visually and expressed in percentages.

What kind of errors and other warning signals can be expected with this task?

A common error observed in both parts of the task is that students identify how many parts are shaded, but then incorrectly report this count as the percentage. For example, in Part A, students might respond with "3" because three out of four parts are shaded—confusing the count with the percentage equivalent. Similarly, in Part B, students often write "4" because four of eight sectors are shaded, overlooking the need to convert the fraction 484⁄884 into a percentage. These errors reflect a surface-level processing of visual data, where students count shaded segments but do not complete the conceptual translation to percentage form. Another frequent issue is that students might not recognize the regularity in Part B, as the alternating shaded sectors obscure the immediate visual impression of "half." As a result, some may misjudge it as less than 50% due to the irregular pattern. These errors indicate a need for stronger skills in bridging concrete and abstract representations, especially in translating between counts, fractions and percents.

What kind of support could be given to children who show deficits with this task?

To support students in mastering this skill, teachers can work with structured visual aids, such as 100-grids, pie charts or fraction bars, to show explicitly how fractional parts relate to percentages. Encouraging students to first name the fraction (e.g., "3 out of 4") before translating it into a percentage helps clarify the intermediate step. It is also helpful to practice with well-known benchmark fractions (e.g., 1⁄2, 1⁄4) and their corresponding percentages. Teachers can model this process out loud: "There are 4 equal parts and 3 are shaded. So it's three-fourths. What is three-fourths as a percent?" Activities involving matching visual models, fractions, decimals and percentages can also strengthen cross-representational understanding. Over time, students can internalize the mappings and become more confident in interpreting visual percentage tasks.

Task 3.4: Calculations with negative numbers

Determine the missing number.

a) $12 - (-5) =$ __17__

b) $11 \cdot$ __(−4)__ $= -44$

Solution

Part a) can be solved by recalling that "minus minus becomes plus" (more exactly: subtracting a negative number gives the same result as adding the corresponding positive number). Based on this principle, the calculation is simple:

$$12 - (-5) = 12 + 5 = 17$$

Part b) can be solved by first noticing that the product -44 is a negative number, while the first factor 11 is a positive number. Hence, the answer to b) has to be a negative number. The corresponding positive number completes the multiplication $11 \cdot$ _____ $= 44$. Since this (positive) number is 4, the answer to b) is -4.

Key skill tested with this task

This task focuses on students ability to reason about the effects of addition, subtraction and multiplication involving negative numbers. It requires them to calculate the task considering the correct interpretation of signs (+ and –).

Why is this skill a key skill?

Understanding the behavior of negative numbers under basic operations is essential for algebraic fluency. This task requires students to apply known sign rule, building the ability to mentally manipulate signed expressions based on structural understanding, not just procedures. Within the DiToM framework, this task supports symbol sense, structural flexibility and conceptual reasoning about operations and order of magnitudes—foundational elements for solving equations, understanding function behaviors and evaluating algebraic expressions.

What kind of errors and other warning signals can be expected with this task?

In part a, many students incorrectly substract 5 from 12, missinterpreting the subtraction signs. In Part b, some students may insert a positive integer without considering the sign rule for multiplication. Alternatively, some might associate substraction only with "smaller" outcomes and fail to reason structurally. Such errors indicate fragile understanding of operation rules with negative numbers, particularly in cases involving nested signs or parentheses. They also show whether students are able to think backward from a result—an essential skill in algebra.

What kind of support could be given to children who show deficits with this task?

Students benefit from tasks that explicitly contrast sign combinations (e.g., (+)(+), (+)(−), (−)(+), (−)(−)) with visual models, such as number lines or colored counters. Teachers can provide structured sentence frames, e.g., "Subtracting a negative is the same as…" and help learners verbalize operations. Having students generate examples where they deliberately vary the signs and observe the result can help internalize the patterns. In Part b, specifically, introducing the multiplication triangle (factor × factor = product) and working backward from the product helps students to deduce missing signs logically. Ultimately, regular practice in reverse tasks—working from results to inputs—can build more robust algebraic thinking.

Task 3.5: Identifying decimal values on a number line

Write a number in the box that represents the position on the number line.

a)
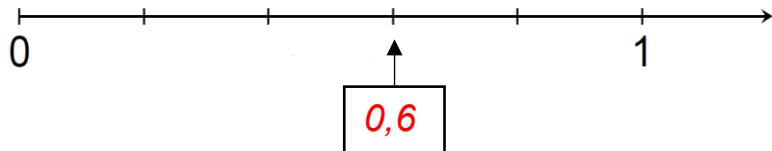
b)
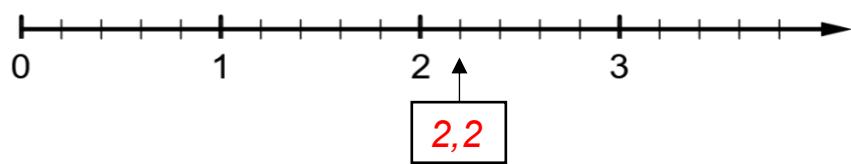
Solution

Two possible strategies:

1) Counting the number of steps (or sub-intervals) from 0 to 1;
2) Testing sequences of decimal numbers (or fractions) from 0 to 1.

First solution to a): In detail, the first strategy can be implemented by counting 5 steps from 0 to 1.

# steps:	1	2	3	4	5
fraction:	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
decimal:	0,2	0,4	0,6	0,8	1,0
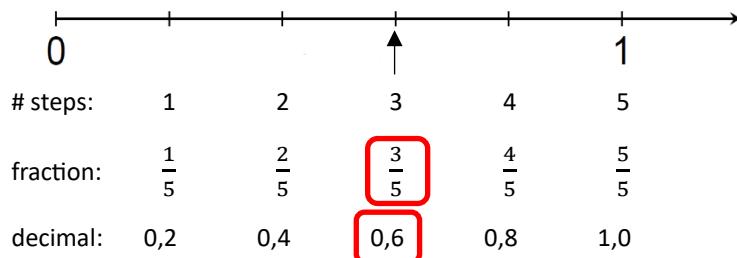
Second solution: One possibility is to test 0,1 for the first position to the right of 0. This choice 0,1 results in 0,5 that does not fit 1 on the number line. Changing to 0,2 results in 1,0 that fits 1 on the number line.

test:	0,1	0,2	0,3	0,4	0,5
adjust:	0,2	0,4	0,6	0,8	1,0
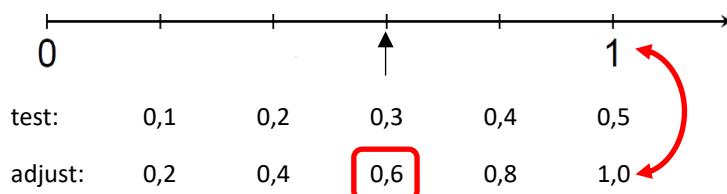
Key skill tested with this task

This task assesses students ability to identify and write decimal numbers based on their position on a number line. Both parts involve interpreting evenly spaced tick marks between whole numbers and assigning the correct decimal value to the indicated position.

This task requires students to combine number line interpretation, decimal understanding and proportional reasoning.

Why is this skill a key skill?

Number lines offer a powerful, visual representation of rational numbers, reinforcing both magnitude and relative positioning. The ability to interpret fractional or decimal positions on a number line is fundamental to understanding equivalence, density and operations with rational numbers. According to Treppo & van den Heuvel-Panhuizen (2014), this task supports the development of mental number sense, rational number representation and the ability to flexibly locate and reason about numbers between whole values. These skills are crucial not only in number theory but also in measurement, data interpretation and algebraic modeling.

What kind of errors and other warning signals can be expected with this task?

Students who struggle with this task often:

- Miscount tick marks, for example mistaking the third mark for 0.3 instead of 0.6 in Part a, by assuming each tick equals 0.1 without verifying the interval based on the number of shares allocated.

- In Part b, students may write 2.25 instead of 2.2, estimating roughly or defaulting to more familiar decimal structures.

- Others might not recognize the increment size, especially when segments don't correspond to tenths and so use incorrect step values.

- A lack of precision when reading off values from graphs is also a warning signal that students are not yet fully comfortable with non-standard partitioning of number lines.

Such errors suggest weaknesses in proportional partitioning, decimal place understanding or in reading visual data carefully.

What kind of support could be given to children who show deficits with this task?

To strengthen this skill, students benefit from hands-on work with drawn and interactive number lines, where they label values themselves and justify placements. Teachers can scaffold the process by asking: "What is the distance between two marks?" and "How many parts is this interval divided into?" Using color coding for each interval or number strips that emphasize uniform partitions can support better estimation and counting. Reinforcing common fraction-decimal equivalences (e.g., 1/4 = 0.25, 1/5 = 0.2) also helps students become more fluent in placing numbers correctly. Lastly, frequent practice with zoomed-in number lines (between 0 and 1 or between 2 and 3) trains students to reason proportionally and to avoid overgeneralizing from whole-number thinking.

VI. Scientific evaluation

This DiToM Screening 8+ was developed on the basis of theory and tested as part of a non-representative validation study. The results reported below serve to identify students who are potentially at risk due to a lack of mathematical key skills for subsequent mathematics learning at school. The test helps teachers at the end of grade 8 / beginning of grade 9, to make an empirically based assessment of students' performance and to identify these with conspicuous results for appropriate support at an early stage.

Description of the sample and central results

The test was trialled from June to July 2025 within the last 3 weeks of the 2024/2025 school year with 1238 students of schools in Greece, Germany, France, Spain, Italy, Croatia and Sweden.

The test comprises the following test parts: Basic Arithmetic Skills with 9 items, Proportionality with 7 items, Technical Calculations with 8 items. If an item was solved correctly, 1 point was awarded; if the solution was incorrect, incomplete or missing, 0 points were awarded. The test was carried out according to standardised criteria (see IV. Implementation of the DiToM test) and evaluated. As the test is designed as a screening test that identifies students who are potentially at-risk, strong ceiling effects were expected (i.e. no normal distribution, but rather a left-skewed distribution) and desired. This was confirmed by the trial.

For practice-oriented communication, not a single threshold value is specified, but two threshold values that differentiate between potentially at risk, students who continue to be observed and students potentially not at-risk. The determination of the cut-off score was data-driven by a latent class analysis with 3 clearly distinguishable classes. The classes are non-overlapping and monotonic. The posterior probabilities of the class assignment were plotted against the point score, smoothed and used to determine the critical point thresholds for funding decisions with regard to their intersection points (see Figure below). The intersection points of the curves were used (posterior probability p=0.5).
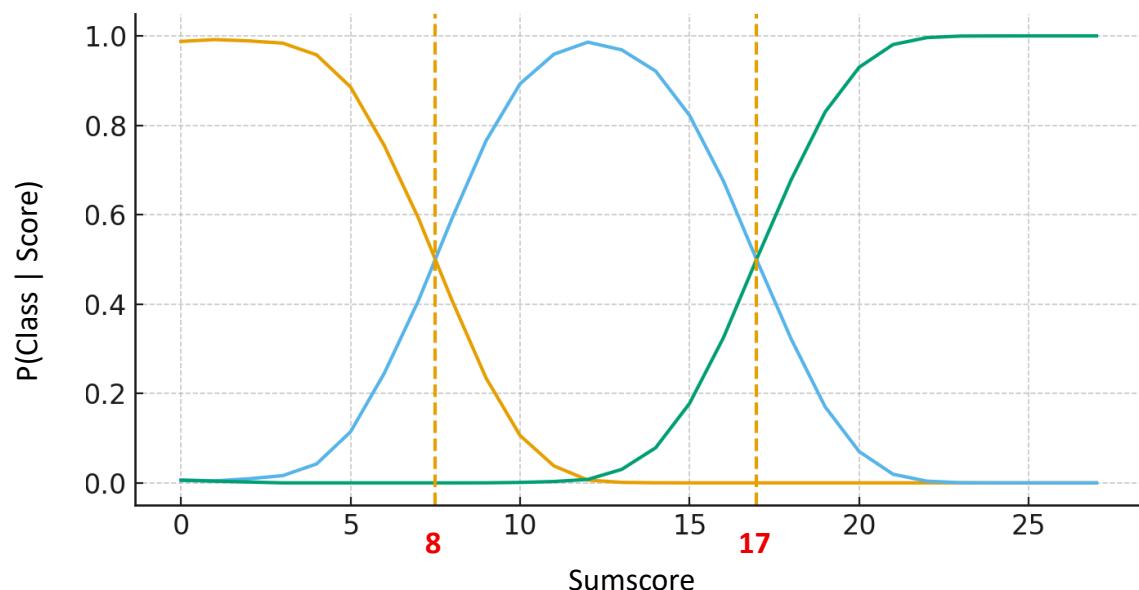
The class analysis shows three clearly distinguishable classes, which are interpreted as K1→ students with weak performance in the screening test, K2→ students with rather weak performance in the screening test and K3→ students with inconspicuous performance in the screening test.

To determine the critical score, the threshold up to which there is a 50% probability of being in the class with poor test performance was selected. This first threshold is therefore 8 points. Students who have achieved a **score**

of 8 points or less need support to work through the basics in order to be able to build up the following content of the maths lessons in an understanding-orientated way. The second threshold is 17 points. Students who have achieved a **score between 9 and 17 points** should be observed in maths lessons over the next few weeks to see whether they understand the content covered and can implement it independently.

VII. Evaluationsheet of Screening 8+

The following scale provides initial indications of the skills with which students most probable score points in the following three ranges: 0-8 points, 9-17 points, and 18-24 points

SCORE

This group is unremarkable in terms of their prerequisites for learning mathematics at school.

Score
24
23
22
21
20
19
18
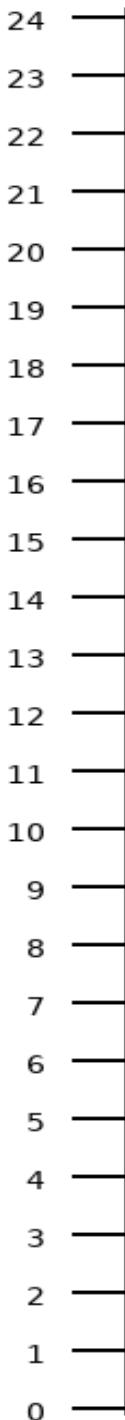
C: In addition to A and B, students are often able to…

… carry out calculations with negative numbers, build a well-structured algebraic expression from a written description, solve division problems in real-world contexts, interpret and work with graphs in proportional situations, translate verbal phrases into algebraic expressions, solve problems involving inverse proportionality and understand and use the perimeter of a rectangle.

And students are rarely able to…

… apply these skills flexibly in complex, multi-step modelling tasks in unfamiliar contexts.

Score
17
16
15
14
13
12
11
10
9
8

This group should be monitored more closely in the coming weeks to see whether the following content can be understood and implemented independently.

B: In addition to A, students are often able to…

… evaluate an expression with variable substitution, convert a fraction to a percentage, build an algebraic expression from a multistep written description, structure a written expression from a multistep rule, identify the solution to a simple linear equation, distinguish between simple directly proportional and non-proportional relationships and identify decimal values on a number line

And students are rarely able to…

… perform calculations involving negative numbers, solve division problems embedded in richer real-world contexts, or to translate more complex verbal descriptions into algebraic expressions.

Score
7
6
5
4
3
2
1
0

This group is at risk in terms of their prerequisites for learning mathematics at school and should receive special support as quickly as possible.

A: Students are often able to…

… read information from a coordinate grid, identify simple directly proportional relationships, interpret circle representations and convert them into percentages, solve simple proportional reasoning problems in a straightforward real-world context, locate simple decimal values on a number line, and calculate a simple percentage increase.

And students are rarely able to…

… evaluate algebraic expressions by substituting values for variables, convert fractions into percentages in more demanding situations, or to build structured expressions from a multistep written description.

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