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DiToM
Diagnostic Tool in Mathematics

Handbook
Screening test 6+

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I. Introduction

Learning mathematics is cumulative: new content builds on secure prior knowledge. If fundamental ideas and concepts are missing, it becomes increasingly difficult for students to construct meaningful understanding of subsequent topics. Findings from international and national studies show that a substantial share of learners do not have minimum standards in mathematics. For day-to-day teaching this means: early, practical procedures are needed to make learning status visible and to organise timely support. This is where the EU project “Diagnostic Tool in Mathematics (DiToM)” comes in. In a partnership of Italy, France, Sweden, Croatia, Greece, Spain and Germany, five interrelated screening instruments have been developed to provide for teachers a compact overview of their class at educational transition points. The screening points follow a two-year rhythm:

1. Transition pre-school → Start at primary school
2. End of Grade 2 / start of Grade 3
3. End of Grade 4/ start of Grade 5
4. End of Grade 6 / start of Grade 7
5. End of Grade 8 / start of Grade 9

What is a screening?
A screening is a short, group-based assessment that can be administered to the whole class within a single lesson. It provides an initial, structured overview of which core ideas are already solid and where individual students may need additional support. A screening, more importantly, does not replace an individual process-orientated, qualitative assessement of a child's current state of mathematical thinking. It serves as a starting point: results can be followed by targeted observations, interviews and support measures.

Why is this helpful?

- Provide a quick overview: which fundamental skills are secure and where review or extension is useful.
- Allow a guided support: identify students who may struggle with minimum standards of basic mathematics; organise early support.
- Make diagnostic decisions: screening results provide a clear first orientation indicating which students may benefit from further diagnostic steps (e.g., deeper task analyses or follow-up interviews).
- Support transitions: focus attention on key skills at crucial school transitions.

The tasks are classroom-oriented, administration is clearly described and scoring is swift. Teachers receive a concise class-level summary plus pointers to which students merit a closer look for specific content areas. On this basis, they can plan short review windows, differentiated practice or bridging tasks.

This handbook provides a compact guide to the purpose and use of the screening instrument, explains the test design, task types and the targeted assessment objectives, gives clear instructions for classroom administration, outlines scoring and interpretation of results and offers practical ideas for subsequent instruction and targeted support.
The aim is a practical, reliable and easy-to-use screening tool that gives teachers quick orientation, draws early attention to potential difficulties and concretely supports effective help so that as many students as possible learn mathematics securely, with understanding and with confidence.

II. What means “mathematical key skills”

Diagnostic test development requires a theoretical basis. For short, whole-class screening tests, this means focusing on those skills without which subsequent content cannot be learned in a meaning-oriented way. Following the classical position of Gagné & Briggs, each new learning demand builds upon a minimal amount of necessary prerequisites what the term mathematical key skills refer to. If these are not available, successful acquisition of the new content is unlikely and appropriate tasks therefore build on what is already in place. In mathematics, learning is accordingly hierarchical and cumulative.

Conceptual understanding: competences, concepts, skills and key skills

Within the project we distinguish between competences and skills, which are mutually dependent in classroom practice. Competences refer to an insightful readiness to act appropriately in mathematical situations. Thereby, concepts capture substantive insight into mathematical relations. The understandable activation of competencies arise in a skill, as practiced performance on side of the students. Key skills are those skills whose absence substantially hinders or prevents further learning. They function as necessary prerequisites for subsequent content. The focus of the screenings are on arithmetic and algebra, due to their hierarchical structure and their importance also for other domains of mathematics (e.g., geometry) which is compatible both nationally and across countries.

The following expand on two examples to clarify the understanding of key skills.

Primary level: executing addition in a structured way
The task 25 + 7 requires more than stepwise counting. A robust operation sense shows when students recognise part–part–whole relations (e.g. 25 and 7 as parts of a whole), decompose numbers flexibly (e.g. 7 = 5 + 2) and build on the next ten (e.g. 25 + 5 = 30; then +2 = 32). Here, concepts (place value, equality), competences (flexible calculation, justified procedure), and the resulting skill (structured addition) work together. If this key skill is missing, the next “level”, larger number ranges or more efficient strategies, remains difficult to access.

Lower secondary: managing extension of number domains
A secured operation sense with natural numbers (decomposition, inverse operations, place-value and number-line references) is a prerequisite for transferring procedures to decimals and fractions (e.g. addition/subtraction, rounding, estimation) to overcome epistemological obstacles involved in learning mathematical concepts (Brousseau, 1997). Gaps in these key skills often lead to procedural work without understanding, which in turn impedes access to algebraic expressions, equations and functional relationships. This illustrates the predictive character of arithmetical key skills for algebraic demands.

The key skill understanding is integrated in the tests to

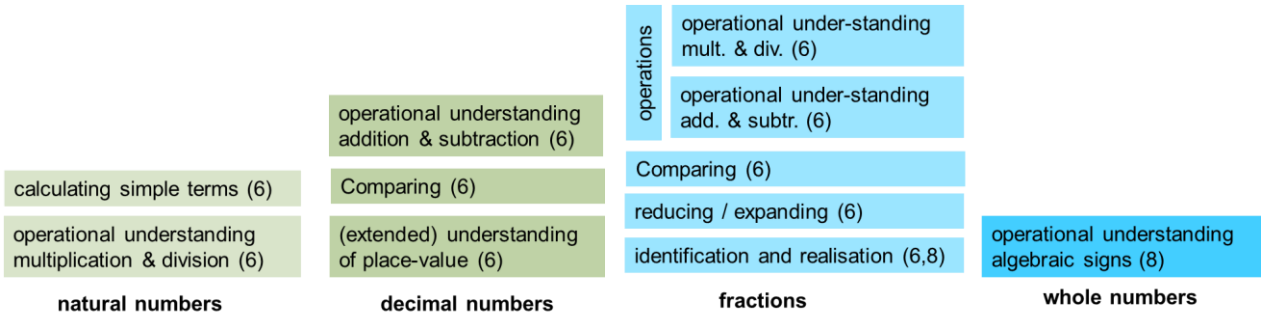
- represent the necessary prerequisites for the next learning step, and
- are content-proximate and thus observable with short tasks, and
- offer teachers a first structured orientation as to which students may require further diagnostic steps and where support can be targeted. The aim is not to assign labels but to reveal central prerequisites early, so that subsequent learning can proceed on a stable basis.

In our understanding, every content domain contains key skills and these may become critical at different points in a learning progression, including at the end of a unit when a capability is needed to enable subsequent learning. The development of key skills is therefore ongoing across grades; identifying missing prerequisites early remains essential so that learners can continue to acquire new content with understanding.

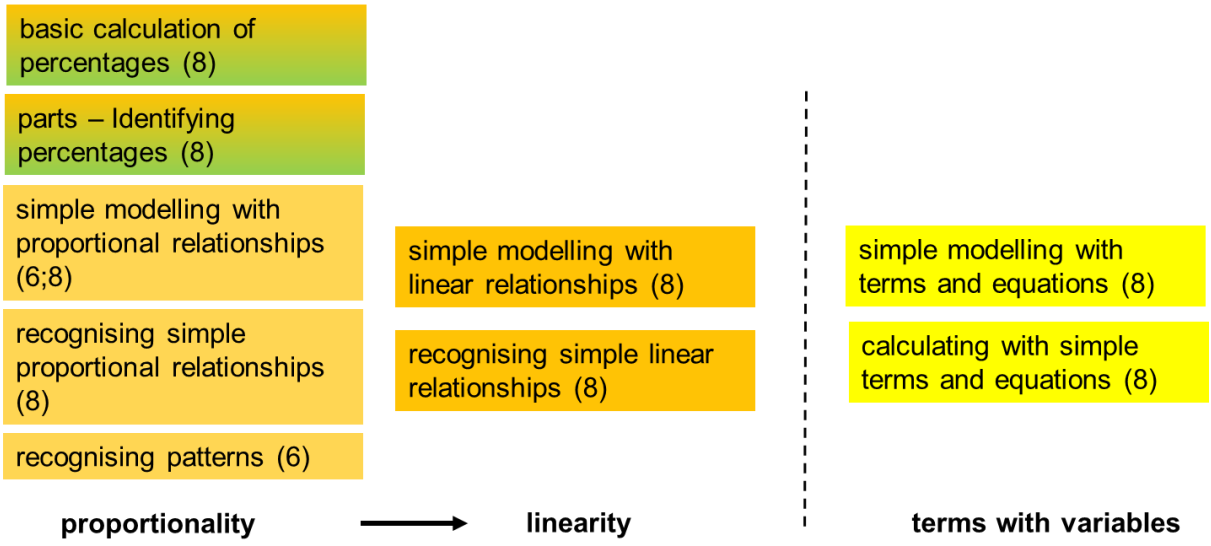
III. Test structure Screening 6+ and 8+

The test structure in DiToM is based on the content areas of arithmetic and algebra. The hierarchical structure of the content area is taken into account. The test construction focussed on the area of number range development and extension in the sense of a technical calculation, insofar as calculation procedures are carried out non-algorithmically and algorithmically on the basis of a fundamental understanding. The diagram shows the test structure in this content area for grades 6+ and 8+.

The test for grade 6+ is based on the building blocks of grade 4+, which focus on natural numbers and this area is more differentiated. If pupils have considerable difficulties in the area of natural numbers in grade 6+, it is recommended to use the test for grade 4+.



In the area of algebra or pre-algebra, the structural understanding of simple mathematical structures in both internal and external mathematical applications is assessed under the aspect of proportionality and linearity. Likewise, in the area of terms with numbers or variables in different directions in basic application situations as well as for term understanding, insofar as it is part of a basic understanding.



IV. Implementation of the DiToM test

- **Explain the purpose of the test to students and reassure them**

    - The test is not graded.
    - It enables them to take stock of what they know and what they don't know, so that they can then suggest appropriate exercises. Therefore, it is particularly important that they work alone.
    - Emphasize the importance of completing the exercises. The more they answer the questions, the easier it will be to identify their knowledge, skills and difficulties, and to help them overcome them.
    - You can also say that this is the first time this test has been used and that the people who designed it want to know if it's suitable.

- **Test structure**

    - The test is divided into three parts, each made up of several exercises.
    - All exercises are independent of each other.

- **Duration:** A maximum duration is estimated for each part.

    - Test grade 6, maximum 45 minutes: 15 for pre-algebra, 10 for proportionality, 20 for arithmetic.
    - Test grade 8, maximum 40 minutes: 15 for pre-algebra, 10 for proportionality, 15 for arithmetic.
    - It is important to indicate the duration of each part to students before the test is taken, and that the teacher will interrupt students who have not finished, out of fairness between students.

- **Exercise format**

    - Open-ended exercises: there's space to answer (either with sentences or with a number).
    - Closed exercises (multiple-choice questions): several answers are proposed, and the student must respond by choosing just one. Please tell the students that if they decide to change their multiple-choice answer, they should write 'No' beside the first answer and 'Yes' beside the new one.

- **How to answer**

    - Calculators are not permitted.
    - Students may use any part of the page left blank as rough draft, in particular to write down their calculations.
    - Students can work through the three parts in order, at their own pace. Students who have completed one part of the test should wait for the teacher to give directions to continue with the next part.

- **Soliciting students during the test**

    - If the teacher is solicited, he/she gives no indication that would guide the answer to the questions. The aim is to identify students' difficulties.

V. Presentation of the tasks

Task 1.1: Multiplication and division

Key skill tested with this task

Find the missing numbers.

a) 3 · ___ = 126

b) 172 = 4 · ___

c) 54 : ___ = 6

d) ___ : 3 = 27

This task targets students understanding of the structural relationship between multiplication and division. Across the four sub-items, students are asked to find a missing number in a task involving either multiplication or division, such as completing “3 · ___ = 126” or “172 = 4 · ___”. To solve these items correctly, students must identify the role of the known and unknown numbers and flexibly move between the operations. They need to interpret the equations not merely as prompts for calculation, but as expressions of a part-whole relationship in which one quantity results from multiplying or dividing two others and recognize equality sign as an equivalence relation. This operational flexibility is a hallmark of deeper arithmetic understanding and is essential for accessing more advanced mathematical content.

Why is this skill a key skill?

Recognizing and working fluently with the inverse relationship between multiplication and division is a key skill for later mathematical learning. This understanding forms the basis for reasoning with ratios, proportions, algebraic expressions and functional relationships. According to the DiToM framework, such skills are classified as mathematical key skills because their absence can hinder or even block future learning progress. Students who can interpret an equation structurally—understanding, for instance, that “3 · ___ = 126” implies “126 : 3”—are demonstrating more than procedural recall: they are engaging in mathematical reasoning. Developing this ability early ensures that students will be better equipped to handle symbolic representations and multi-step problem-solving in secondary mathematics.

What kind of errors and other warning signals can be expected with this task?

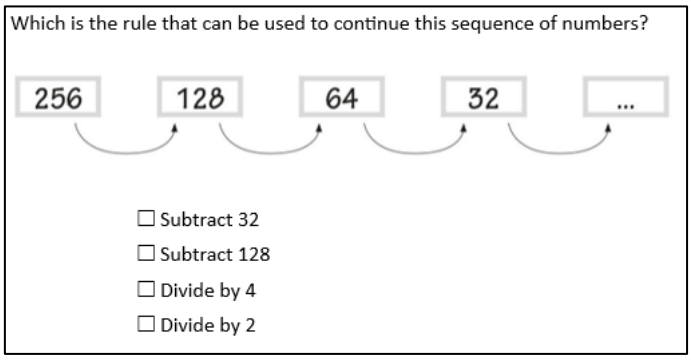
Students who have not yet internalized the relationship between multiplication and division often display frequent misconceptions. A common error is interpreting all equations as requiring forward multiplication, even when the inverse operation is needed. For example, when encountering “172 = 4 · ___”, students may mistakenly calculate “172 · 4” instead of dividing. Others may guess based on rote fact recall, without considering the structure of the equation. Misunderstanding the function of the equal sign—as a cue to calculate rather than a symbol of equivalence—can also lead to procedural but incorrect answers. In some cases, students attempt complex written methods (such as long division, e.g. 126=120+6=3x40 + 3x2 or 126:3=40+2=42) where a strategic understanding of number relationships would be more appropriate. These behaviors can indicate a lack of structural awareness, as well as limited conceptual fluency.

What kind of support could be given to children who show deficits with this task?

It is important to explicitly connect related number facts (e.g., “6 · 4 = 24”, “24 : 4 = 6” and “24 : 6 = 4”) to highlight the reversibility of the operations. Encouraging students to verbalize their reasoning, for example by asking “What is the number that, when multiplied by 4, gives 172?”, supports internalization. Finally, variation in the position of the unknown (beginning, middle, or end of the equation) should be practiced to deepen flexible understanding of equation structure.To support students who struggle with this concept, it is helpful to work with visual representations such as arrays, bar models or grouping diagrams that make multiplicative structures visible (Polotskaia & Savard, 2021). These models allow students to see how a quantity can be composed of equal parts or decomposed into them—mirroring multiplication and division, respectively.

Task 1.2: Number patterns and rule identification

Key skill tested with this task



This task assesses students' ability to identify and describe the underlying rule in a numerical sequence. The specific example—256, 128, 64, 32, …—requires recognizing a geometric progression in which each number is the result of dividing the previous one by two. The task presents multiple choice options, prompting students to decide which rule (e.g., "subtract 32" or "divide by 2") correctly explains the pattern. Thus, the core skill being tested is students capacity to recognize multiplicative structures. This involves more than procedural knowledge—it requires pattern recognition, structural reasoning and early algebraic thinking.

Why is this skill a key skill?

The ability to recognize regularities in number patterns is a key mathematical skill because it forms the basis for more advanced concepts such as functions, algebra and proportional reasoning. Students who can discern rules in sequences are better equipped to engage in generalizations and symbolic representations later on. According to research in mathematics education (e.g., Kieran, 2018, Radford 2013), pattern recognition supports the development of a relational understanding of numbers and operations. Within the DiToM framework, the identification of numerical structure is seen as essential for navigating the increasing abstraction of secondary mathematics. Moreover, understanding geometric sequences—such as halving—also lays important groundwork for interpreting exponential relationships, a concept encountered in later grades.

What kind of errors and other warning signals can be expected with this task?

A frequent misconception in this task is interpreting the sequence as additive rather than multiplicative. Students might assume the numbers decrease by a fixed amount and select "subtract 32" because the difference between 64 and 32 fits this pattern, even though it does not apply consistently to earlier steps. Such errors reveal a linear bias, which is common when students are unfamiliar with geometric change. Other students may guess without checking the pattern across multiple terms, demonstrating unsystematic reasoning. Additionally, if students do not yet see division as the inverse of multiplication, they may not recognize "divide by 2" as a recurring structure. These patterns of error suggest a fragile or underdeveloped concept of operational structure and sequence.

What kind of support could be given to children who show deficits with this task?

Students who struggle with recognizing numerical patterns benefit from structured tasks that explicitly contrast additive and multiplicative relationships. Using visual aids such as number chains or tree diagrams can help students identify how values change from step to step. Activities that require students to generate their own sequences from given rules (e.g., "Create a sequence where each number is half the one before") can build pattern awareness and deepen operational understanding. Teachers should encourage reasoning aloud—for example, "How did the number change from 256 to 128?"—to promote metacognition and make strategies visible. Over time, connecting these patterns to real-world contexts (e.g., folding paper, doubling bacteria) can reinforce the concept of geometric progression and make abstract patterns more tangible.

Task 1.3: Order of operations (priority rules)

Key skill tested with this task

Calculate:
$14 + 2 \cdot 3 =$ _____

This task tests students understanding of the conventional order of operations, specifically the priority of multiplication over addition. Students must correctly interpret and evaluate the expression "14 + 2 · 3", applying the rule that multiplication is performed before addition. This requires not only procedural fluency but also an awareness of the hierarchical structure of arithmetic operations. The task therefore goes beyond factual knowledge and assesses students ability to parse and structure numerical expressions correctly—a crucial step toward algebraic literacy.

Why is this skill a key skill?

Understanding operation order is a foundational prerequisite for working with more complex arithmetic expressions and, later, algebraic expressions and equations. Within the DiToM framework, the ability to process multi-step expressions according to mathematical conventions is considered a key skill because it underlies symbolic reasoning and general problem-solving ability. Students who internalize these rules can interpret expressions reliably, manipulate terms confidently and avoid common pitfalls in calculations. This competence is not only essential in the context of number operations but also directly transferable to expression and formula work, equation solving and function analysis in later grades.

What kind of errors and other warning signals can be expected with this task?

A typical error in this task is evaluating the expression from left to right without respecting the hierarchy of operations—i.e., adding 14 and 2 first to get 16 and then multiplying by 3 to obtain 48. This mistake reveals a linear computation bias and a lack of conceptual understanding of operation precedence. Another warning sign is hesitation or overreliance on informal reasoning strategies ("just do what's written first"), suggesting that students may be applying everyday reading order rather than mathematical structure. Some students might also attempt to insert parentheses inappropriately, showing insecurity about how expressions are organized. Even if students arrive at the correct answer, the use of "trial-and-error" or guesswork rather than structured reasoning can be a signal of conceptual gaps.

What kind of support could be given to children who show deficits with this task?

Targeted support should begin by making the structure of expressions visible, for instance through the use of color coding, brackets or visual models that show grouping. Teachers can model the evaluation of expressions step by step and encourage students to verbalize their reasoning: "First, I do 2 times 3 because multiplication takes precedence over addition. Then I add 14." Similarly, iconic representations of the two calculation methods can help to distinguish and grasp the calculation priorities. Practice with a variety of expressions—including those with and without parentheses—can help clarify when and why order matters. Students also benefit from exploring incorrect strategies and discussing why they lead to wrong results. Over time, regular exposure and structured reflection help internalize the rules and strengthen students confidence in handling multi-step calculations.

Task 1.4: Translating written text into mathematical expressions

Key skill tested with this task

This task evaluates students ability to interpret a short verbal sequence describing two consecutive operations—first an addition, then a multiplication—and translate this sequence into a symbolic expression. Students are not expected to calculate the result but to identify the correct mathematical representation of the instructions because verbal instructions and the corresponding numerical expression are not semantically congruent (Vergnaud, 1983). This requires recognizing the order of operations embedded in the language and constructing a term accordingly (e.g., $(4 + 5) \cdot 8$). The core skill tested is the translation from natural language to formal notation, including the use of parentheses to preserve the correct computational structure and operational priorities.

Tom follows the instructions:

The number 4 is added to 5.
The result is multiplied by 8.

What calculation can Tom use to obtain the result?

- ☐ $5 + 4 \cdot 8$
- ☐ $(5 + 4) \cdot 8$
- ☐ $5 + (4 \cdot 8)$
- ☐ $5 \cdot 8 + 4$

Why is this skill a key skill?

The ability to represent verbal or contextual information symbolically is central to mathematical literacy. Within the DiToM framework, this skill is considered a key skill because it enables students to move between different representational modes—verbal, symbolic, iconic and operational and dealing with the structure sens (Kieran & Martìnez-Hernàndez, 2022) of numerical expressions. This translation competence is essential not only in arithmetic but also in algebra, where students regularly encounter situations that require building or interpreting expressions from word problems, diagrams or everyday scenarios. Early mastery of this skill supports the development of functional thinking, problem-solving flexibility and fluency in working with mathematical models.

What kind of errors and other warning signals can be expected with this task?

A common error in this task is constructing the expression in the wrong order, such as interpreting “4 is added to 5” as “4 + 5” (what is mathematicaly correct) but then applying multiplication incorrectly: either “4 + (5 · 8)” or “4 · 5 + 8”. This reflects difficulty identifying the sequence of operations embedded in language. Some students may ignore the need for parentheses, writing “4 + 5 · 8”, which leads to an incorrect order of operations if computed. Others might focus only on the final operation and write “9 · 8 = 72” as the answer, bypassing the actual task of symbolic translation. These patterns suggest gaps in procedural understanding and difficulties coordinating language with mathematical structure.

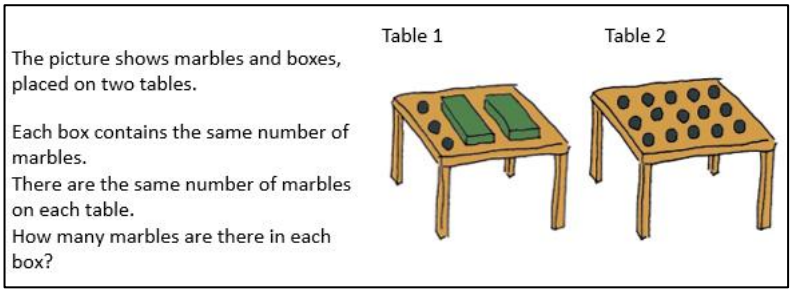
What kind of support could be given to children who show deficits with this task?

To support students in this area, it is important to encourage students to develop a comprehensive understanding of the sequence of calculations in order to take into account the structure of the calculation and the order of operations.. Teachers can model how to “build a term” from a spoken sentence and use visual organizers (such as operation trees or flowcharts) to help students sequence the operations correctly. Emphasizing the role of parentheses in grouping operations can prevent misinterpretation. Classroom routines that involve “translating back and forth” between language and symbols can also strengthen students representational flexibility. Over time, encouraging students to say what the expression means (e.g., “first I add, then I multiply”) helps consolidate their understanding of symbolic structure.

Task 1.5: Equating quantities

Key skill tested with this task

This task assesses students ability to interpret a visual representation of a part–whole situation involving equality. Students are shown two tables, each containing a combination of visible marbles and boxes that each hide the same unknown number of marbles. The key requirement is to deduce the number of marbles in one box based on the information that both tables contain the same total number of marbles. This means students must mentally equate the quantities on both sides and solve for the unknown—a form of informal equation solving based on visual balance. The task therefore targets structural reasoning, early algebraic thinking and the ability to interpret equivalence in a non-symbolic context.



Why is this skill a key skill?

Interpreting equality is a critical precursor to algebraic reasoning. Within the DiToM framework, this is identified as a key skill because it taps into students understanding of equivalence and substitution—central ideas in both arithmetic and early algebra. By reasoning that two different configurations must in fact be equal in total, students are practicing relational thinking rather than relying on direct computation alone (Radford, 2014). This skill supports later competence in equation solving, balancing transformations and working with unknown quantities in symbolic form. Moreover, such non-symbolic tasks provide an essential bridge for students who are still developing confidence with formal representations, enabling conceptual access through visual structure.

What kind of errors and other warning signals can be expected with this task?

Students who struggle with this task may fail to recognize the equivalence between the two sides. A typical mistake is attempting to count only visible marbles, ignoring the hidden quantity in the boxes or assuming a fixed value (e.g., “each box must have 10 marbles”). Others may recognize the need for balance but miscalculate or misalign their reasoning, perhaps guessing the number in a box without checking that it leads to equal totals. Another group of students might treat the visual image descriptively rather than analytically—reporting what is visible without attempting to infer the unknown. These behaviors suggest gaps in structural understanding, in particular in interpreting unknowns as quantities to be determined through inferences made starting from the known quantities.

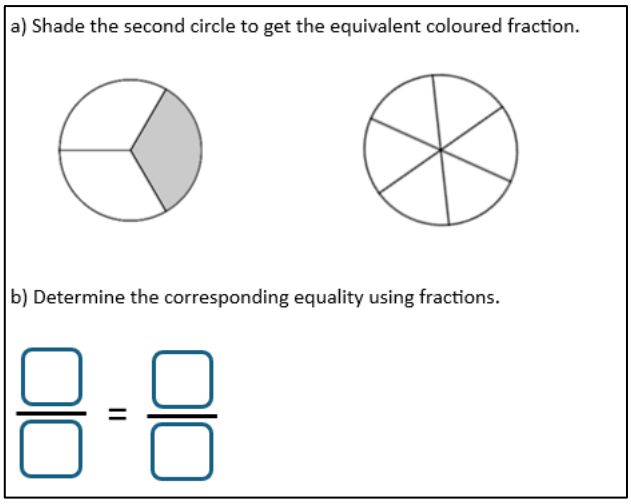
What kind of support could be given to children who show deficits with this task?

Students benefit from working with hands-on materials that make the concept of equivalence concrete—e.g., to advance to the following grades with relative numbers. Teachers can use storytelling contexts (“Both children got the same number of marbles—how many are in the box?”) to support engagement and anchor the problem in a relatable setting. Drawing and labeling diagrams where students write equations like “3 + x = 7” can help bridge visual reasoning and symbolic representation. Additionally, repeated practice in identifying equal but differently composed sets reinforces the notion of equivalence and supports the shift from additive reasoning to early functional thinking. As always, students should be encouraged to explain their reasoning and to check whether their proposed values maintain balance.

Task 2.1: Representing and interpreting equivalent fractions

Key skill tested with this task

This task focuses on the ability to recognize and construct equivalent fractions across two representational aspects: first in a visual format (shading parts of a circle) and then in symbolic notation (writing a fraction equality). In part (a), students are asked to complete a visual representation by shading the same proportion of a circle as shown in a given model. In part (b), they are expected to express this relationship as a mathematical identity using fractions. The core skill tested is the coordination between visual part-whole understanding and its formal representation as equivalent numerical proper fractions.



Why is this skill a key skill?

Understanding equivalent fractions is a cornerstone of rational number understanding and thus constitutes a mathematical key skill. It forms the conceptual foundation for operations with fractions, proportional reasoning, ratio concepts and algebraic equivalence. Within the DiToM framework, recognizing that different-looking fractions can represent the same quantity is seen as crucial for developing flexibility in number thinking. Students must grasp that a proper fraction does not only represent a number, but also a relationship between a part and a whole—and that this relationship remains constant even when both numerator and denominator are scaled. Tasks that combine visual and symbolic levels promote deeper understanding and support transitions to abstract reasoning in later mathematics.

What kind of errors and other warning signals can be expected with this task?

Students may shade an incorrect number of parts in the second circle—e.g., matching the number of a shaded piece rather than the proportional size. This reveals a counting strategy rather than relational thinking, indicating that they see the numerator as a static number rather than as part of a whole. In part (b), students might copy the given fraction without transformation, write non-equivalent but similar-looking fractions (e.g., doubling only the numerator), or confuse the order of numerator and denominator. Some may omit the equality sign entirely, suggesting uncertainty about fraction notation conventions. These are warning signs of fragile conceptual understanding and limited experience in connecting visual and symbolic representations.

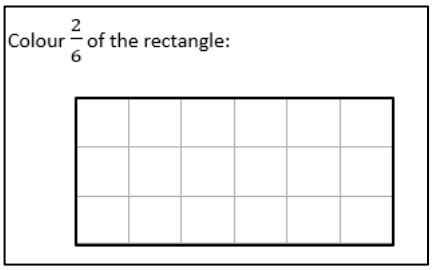
What kind of support could be given to children who show deficits with this task?

To build a solid understanding of equivalent fractions, students should regularly work with manipulatives and visual models—such as fraction circles, bars, or tiles—to see and create equal parts across different divisions. Emphasis should be placed on identifying how many parts of how many make up the same proportion and how both the number of shaded parts and the total number of parts change in parallel. Teachers can guide students in verbalizing the scaling process, e.g., “I doubled the number of parts and doubled how many were shaded.” This supports internalization of the multiplicative structure behind equivalence. Bridging activities—e.g., shading, then writing, then verbally explaining—are particularly effective in stabilizing the link between visual images and formal fraction equations.

Task 2.2: Shading a given fraction of a rectangle

Key skill tested with this task

This task assesses students ability to construct a visual representation of a given fraction by shading a specified part of a rectangular area. Students are expected to identify the correct number of equal parts and shade the number of parts corresponding to the numerator, while recognizing that the total number of parts corresponds to the denominator. This requires interpreting fractions as operators on areas—i.e., using a fraction to define how much of a whole region is being considered. The task demands accurate subdivision, spatial estimation and proportional reasoning.



Why is this skill a key skill?

Constructing a fraction visually is a key step in developing relational and proportional thinking, as well as bridging between informal and formal fraction knowledge. Within the DiToM framework, this skill is considered fundamental because it supports later understanding of equivalence, addition and subtraction of fractions and area-related reasoning in geometry. Representing fractions in a visual model such as a rectangle also reinforces the understanding that fractions are not simply about discrete parts (like marbles or counters), but also about continuous quantities and areas. Students who can flexibly move between fraction notation and visual models tend to develop deeper, more connected number concepts and are better prepared for abstract work in algebra and beyond.

What kind of errors and other warning signals can be expected with this task?

Common errors include shading an incorrect number of parts, often due to miscounting or misidentifying the total number of subdivisions. Students may shade parts that are not equal in size, thereby violating the requirement that fractional parts must be of equal area. Others may shade randomly without establishing any relation to the given fraction, indicating a lack of part-whole conceptualization. In some cases, students ignore the denominator and simply count units (e.g., shading two parts regardless of how many total there are). These patterns point to difficulties in coordinating the symbolic fraction with the visual model and in understanding the structural constraint that defines a valid fraction.

What kind of support could be given to children who show deficits with this task?

Targeted support should include hands-on activities with fraction strips, paper folding, or grid-based area models. Students should be encouraged to first divide shapes into equal parts before applying the operator (e.g., “Shade 3 out of 4 equal parts”). It helps to model examples and non-examples—e.g., rectangles where parts are not equal—to clarify what counts as a valid fraction representation. Linking shading tasks with symbolic writing and verbal explanation (“I split it into 6 equal parts and shaded 4 of them, so that’s four sixths”) supports the integration of representations. Over time, students should practice with varying shapes and orientations to generalize their understanding beyond specific formats.

Task 2.3: Proportional reasoning with quantities and prices

Key skill tested with this task

2 kg of potatoes cost 5 €. Determine the price for 6 kg of potatoes.

This task assesses students ability to apply multiplicative reasoning to solve a proportional problem involving prices and quantities. The context—determining the price for 6 kilograms of potatoes given that 2 kilograms cost 5 euros—requires students to recognize and maintain a constant ratio between quantity and price. To solve this correctly, students must either scale the quantity-price pair by a factor of 3 or calculate the unit rate (price per kilogram) and then multiply. The skill being tested is the understanding and application of multiplicative structures in functional relationships, which is a key foundation of ratio, proportion and percentage problems in later mathematics.

Why is this skill a key skill?

Proportional reasoning is among the most important mathematical key skills in secondary education. According to the DiToM framework, the ability to identify and work with constant relationships—such as "2 kg → 5 €" scaled up to "6 kg → ? €"—is critical not only in arithmetic, but in algebra, functional understanding, geometry, science and everyday problem solving. Students who master these multiplicative relationships can generalize across contexts and flexibly choose efficient strategies (e.g., doubling, halving, unit price reasoning). Moreover, the transition from additive to multiplicative comparison reflects a developmental leap in mathematical understanding that underpins future learning in linear functions and proportional models.

What kind of errors and other warning signals can be expected with this task?

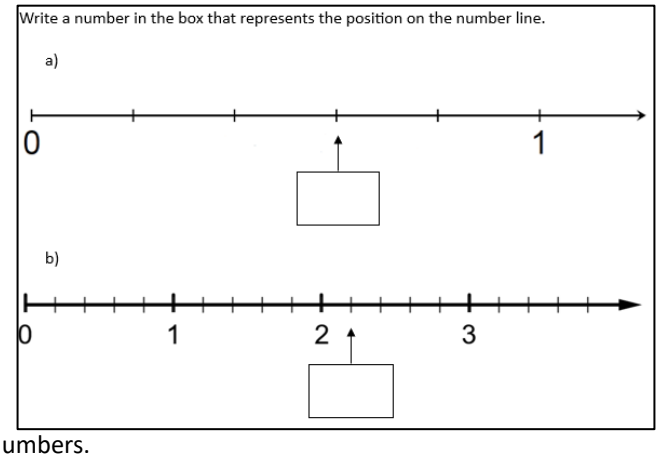
Common errors include additive reasoning, such as assuming that if 2 kg cost 5 euros, then 6 kg must cost 5 + 4 = 9 euros. This reflects a failure to grasp the multiplicative nature of the relationship. Some students may multiply 5 by 6 directly (resulting in 30 €), misinterpreting the meaning of the numbers involved. Others may struggle to coordinate units—mixing kilograms and euros—or simply guess based on estimation. These mistakes point to gaps in structural understanding and possibly limited experience with ratio-based reasoning. Students who do not articulate their strategy or who rely on trial-and-error often lack a reliable conceptual model for proportionality.

What kind of support could be given to children who show deficits with this task?

Students benefit from context-rich problems involving money, recipes, or measurements, where proportional structures naturally occur. Teachers should explicitly model strategies such as unit rate thinking ("If 2 kg cost 5 €, then 1 kg costs 2.50 €…") or factor-based scaling ("6 kg is 3 times 2 kg, so the price is 3 · 5 €"). Visual aids like double number lines, ratio tables and bar models can make the multiplicative relationship concrete. It's also helpful to contrast additive and multiplicative strategies in class discussions to highlight their different implications. Encouraging students to explain and justify their reasoning supports metacognitive growth and helps deepen understanding of proportional structures.

Task 3.1: Symbolically representing numbers on a number line

Key skill tested with this task



This task targets students' ability to interpret a number line segmented into sub-intervals and to place a fraction or decimal number appropriately based on its relative position between 0 and 1 (or beyond). Students must analyze the divisions of the line, determine the unit and identify the correct fraction or decimal number that marks a given point. This requires an understanding of fractions as numbers with magnitudes, not just part-whole relationships. The task also tests the ability to coordinate symbolic and spatial representations of rational numbers.

Why is this skill a key skill?

Being able to locate fractions on a number line is a critical mathematical key skill because it reflects a shift in understanding from fractions as parts of objects to fractions as numbers on a continuous scale. This spatial interpretation of fractions lays the groundwork for comparing, ordering and computing with fractions. Within the DiToM framework, number line estimation and positioning are considered strong indicators of conceptual clarity. Research (e.g., Siegler & Booth, 2004; Treppo & van den Heuvel-Panhuizen, 2014) shows that students who understand the metric structure of the number line are more likely to succeed in later arithmetic, algebra and geometry. Moreover, the number line offers a unified model that supports transitions between natural numbers, fractions, decimals and negative numbers.

What kind of errors and other warning signals can be expected with this task?

Students often rely on counting tick marks supposing a decimal subdivision of the unit rather than reasoning about fractional size. For example, they may misinterpret four divisions as “fourths” regardless of whether the whole is subdivided into equal parts or not. Another common mistake is placing the fraction at the wrong location—e.g., misplacing 3/4 at 2/3 because of a lack of proportional reasoning. Some students may guess based on visual intuition rather than calculating the denominator implied by the divisions. In more advanced variations, students may struggle when the number line does not begin at 0 or when improper fractions or mixed numbers are involved. These errors point to insufficient integration of magnitude, notation and structure. There are also students who give an incorrect decimal number as the solution. Please refer to task 3.4 which addresses this issue.

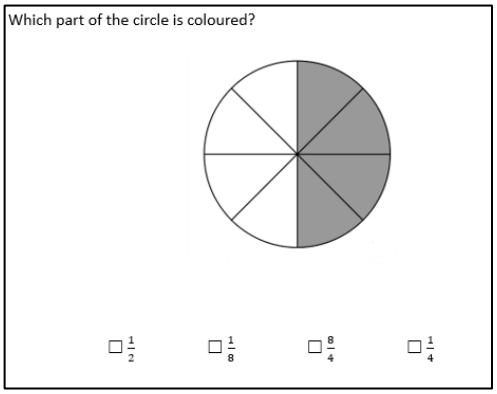
What kind of support could be given to children who show deficits with this task?

To support students, it is essential to spend time building a strong mental model of the number line that includes fractions and decimal numbers. Teachers can use interactive tools such as folding strips, fraction rulers and digital number lines to develop proportional reasoning. Explicit teaching should focus on how to determine the size of one unit interval, how to count fractional steps and how to relate these steps to the written symbol. Comparing different fractions on the same line helps reinforce relational magnitude and equivalence. Bridging activities—like drawing fractions on a line, then writing them in symbolic form and vice versa or establishing relations with iconic representations already known by the students (e.g., circle diagrams)—strengthen representational connections. Frequent verbalization (“This is the third segment out of four, so it's three fourths”) promotes internalization of the structure.

Task 3.2: Choosing the correct fraction of a shaded circle

Key skill tested with this task

This task targets students ability to identify a fraction based on a visual part–whole representation and to select the correct symbolic representation of that fraction from multiple options. In the image, a circle is divided into eight equal parts and four of these parts are shaded. The correct fraction is thus 4/8, which simplifies to 1/2. However, students must not only recognize this relationship but also distinguish it from plausible but incorrect distractors, such as 1/4, 1/8, or even 8/4. The core skill being tested is the coordination between visual, numerical and structural understanding of fractions.



Why is this skill a key skill?

Being able to interpret fractions from visual models and to correctly map this to symbolic representations is a foundational skill in rational number understanding. Within the DiToM framework, this task taps into the concept of fractions as ratios of part to whole, which is central to more advanced concepts such as equivalence, fraction operations and proportionality. Importantly, the task introduces a conceptual trap—the distractor 8/4 is numerically larger than the whole, despite matching the correct numbers (just reversed). Recognizing this mismatch requires more than visual counting; it demands an understanding of fraction structure, scale and meaning.

What kind of errors and other warning signals can be expected with this task?

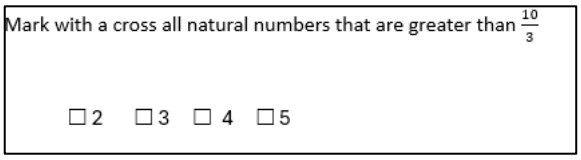
The distractor 8/4 is especially attractive because it includes the two numbers present in the image—8 parts in total and 4 shaded—but reverses their order. Selecting 1/8 or 1/4 could indicate misinterpretation of the proportion, either by counting only the shaded or failing to account for the total. Some students may default to familiar "benchmark fractions" like 1/4 or 1/2 without analysis. These are all warning signals of fragile or incomplete fraction understanding, particularly with regard to part–whole coordination and symbolic interpretation.

What kind of support could be given to children who show deficits with this task?

Students benefit from hands-on activities using fraction circles or folding paper shapes, where they can physically divide and shade parts of a whole. Teachers should emphasize the roles of numerator and denominator through consistent verbalization ("4 parts shaded out of 8 equal parts – that's 4 eighths … and I write it in this way: 4/8"). Practice in matching visual models with multiple fraction expressions, including those greater than 1, can help students distinguish between proper, improper and equivalent fractions. Encouraging students to explain why a fraction like 8/4 cannot represent less than a whole fosters critical thinking and supports structural awareness. Highlighting common errors through guided discussion (e.g., "Why might someone think 8/4 is correct?") can help make misconceptions explicit and address them directly.

Task 3.3: Comparing an improper fraction with natural numbers

Key skill tested with this task



This task assesses students ability to compare a non-unit fraction greater than 1 (10/3) with several natural numbers. Students must identify all the numbers in the set {2, 3, 4, 5} that are greater than 10/3. Since 10/3 is equal to approximately 3.33, the correct solution is to select both 4 and 5. Importantly, the item is scored as correct only if both values are selected and none of the incorrect options are ticked.

Why is this skill a key skill?

Comparing fractions to whole numbers is a mathematical key skill because it links rational and whole number systems, supporting the development of a coherent number line model. Within the DiToM framework, this comparison promotes an understanding of fraction magnitude, estimation and the transition between fractional and decimal or mixed representations. The ability to determine whether a fraction is greater or smaller than a whole number is essential for developing flexibility in interpreting numerical information. Moreover, this skill supports success in tasks involving measurement, scaling and function interpretation—areas where rational–natural comparisons regularly occur.

What kind of errors and other warning signals can be expected with this task?

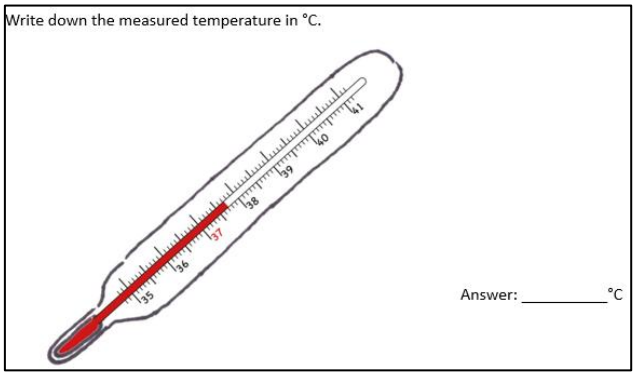
Students might convert 10/3 incorrectly, e.g., estimating it as 2 or 5, leading to inaccurate tick-box decisions. A common misconception is to focus only on the numerator and denominator in isolation—e.g., assuming that 10/3 is smaller than 4 because “3 is greater than 1.” Some students may tick only one correct option (e.g., 4), misunderstanding the task’s instruction or failing to recognize that more than one correct value exists. Others might tick all numbers greater than 3, guessing based on partial recall. These errors reveal a lack of fluency with improper fractions and difficulties in reasoning flexibly between representations (e.g., converting 10/3 into 3⅓).

What kind of support could be given to children who show deficits with this task?

Students should regularly practice comparing improper fractions with both natural numbers and mixed numbers. Visual tools like number lines or fraction strips can help clarify where a given fraction lies relative to benchmark numbers. Teachers can guide students to express improper fractions in mixed number form (e.g., 10/3 = 3⅓) to support estimation and comparison. Exercises involving verbal reasoning (“Is 10/3 more or less than 4?”) and explanation tasks (e.g., justifying why 3 is not correct) help promote conceptual clarity. In tasks with multiple correct answers, it's also helpful to emphasize task literacy—how to interpret "tick all that apply" structures precisely and completely.

Task 3.4: Reading decimal numbers from a Thermometer

Key skill tested with this task



This task examines students ability to interpret and read decimal numbers on a graduated scale, embedded in a real-life context (a thermometer). Students are shown an analog thermometer with Celsius markings and a red liquid column rising to a specific level—namely, 37.7 °C. To solve the task correctly, students must determine the temperature reading from the given visual representation and express it in decimal notation. The core skill tested here is the ability to accurately read and interpret decimal quantities on a continuous, metric scale in a familiar setting.

Why is this skill a key skill?

Within the DiToM framework, interpreting measurements on linear scales is a foundational mathematical key skill because it integrates place value understanding, magnitude estimation and metric reasoning. Decimal number interpretation is essential for everyday contexts—temperature, money, length, weight—and it builds the basis for further work in percentages, fractions and functions. Moreover, reading scales in authentic contexts supports mathematical literacy, as students must make sense of graduated measurement tools in health, science, or daily life. The connection between visual information and numerical representation strengthens students ability to link continuous quantities with symbolic precision.

What kind of errors and other warning signals can be expected with this task?

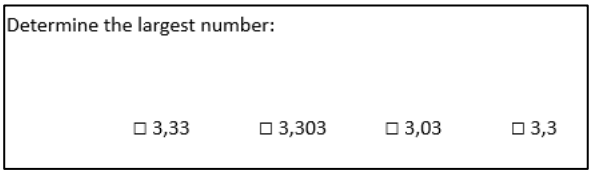
Some students may struggle to interpret fine subdivisions on the scale, especially if the increments are tenths (0.1 steps) rather than whole numbers. Common errors include rounding to the nearest whole number (e.g., 38 instead of 37.7), omitting the decimal place (writing 377), or miscounting tick marks due to unfamiliarity with decimal structure. Others may confuse the value of each interval—e.g., assuming the distance between 37 and 38 is divided into 5 instead of 10 equal parts. These errors often stem from insufficient place value understanding, limited decimal number fluency, or a lack of experience interpreting measurement scales.

What kind of support could be given to children who show deficits with this task?

Students benefit from repeated exposure to scaled measuring instruments, such as thermometers, rulers and graduated cylinders. Teachers should model how to analyze intervals, determine the increment size and count forward using decimal tenths. Using transparent overlays or colored markers to track liquid levels can improve visual alignment. Students should also practice reading and writing decimal numbers in context-rich situations, supported by number line models that bridge symbolic and visual thinking. Emphasizing language precision ("three tenths more than thirty-seven") can help strengthen conceptual clarity around decimal place value. Lastly, targeted tasks comparing values like 37.7, 37.8 and 38.0 can improve fine-grained decimal discrimination.

Task 3.5: Comparing decimal numbers

Key skill tested with this task


Determine the largest number:

☐ 3,33 ☐ 3,303 ☐ 3,03 ☐ 3,3

This task assesses students ability to compare and order decimal numbers, especially those that are close in value and vary in the number of decimal digits. Students must determine which of the four given decimals (3.33, 3.303, 3.03, 3.3) is the greatest. To succeed, they must understand that place value determines magnitude, not the number of digits or the apparent "length" of the decimal. The task specifically tests precision in interpreting on a number line to the hundredths and thousandths place and the ability to recognize that 3.33 is greater than 3.303, despite the latter having more digits.

Why is this skill a key skill?

Comparing decimal numbers is a mathematical key skill because it reflects an understanding of place value in the base-ten system beyond whole numbers. Within the DiToM framework, this is crucial for developing competence in estimation, measurement and real-world numeracy (e.g., prices, data interpretation). Decimal number comparison also supports later work in percentages, algebra and scientific contexts. The task reflects how students reason about relative magnitude and whether they focus on value rather than surface structure, such as digit count or visual length of segments. This skill is foundational for both mental calculation and interpreting tabular or graphical data.

What kind of errors and other warning signals can be expected with this task?

A frequent misconception is that longer decimal numbers are larger—e.g., students may incorrectly choose 3.303 because it has three decimal digits. Others may compare only the first digit after the decimal and neglect the structure beyond (e.g., assuming 3.3 > 3.33 because 3.3 has only one decimal place). Some students may be unsure how to align decimal places and mentally compare the values, especially when they have different lengths. These responses indicate fragile understanding of decimal place value, particularly in distinguishing tenths, hundredths and thousandths. Errors may also reflect inexperience with close-decimal comparisons where intuitive strategies fail.

What kind of support could be given to children who show deficits with this task?

Students should be encouraged to use place value charts to align and compare decimals digit by digit. Teachers can model strategies like adding trailing zeros to equalize the number of decimal places (e.g., comparing 3.300, 3.330, 3.303). Visual tools such as number lines with decimal markers, base-ten block models for decimals, or grid representations can help solidify magnitude understanding. Emphasizing value over appearance ("More digits doesn't mean more value") is critical. Practice with reasoning tasks ("Which is bigger and why?") and estimation games involving money, length, or volume can help embed decimal comparison in meaningful contexts.

Task 3.6: Finding missing addends in decimal equations

Key skill tested with this task

Determine the missing number.

a) 1,8 + ____ = 5,3

b) ____ + 0,51 = 2

This task assesses students ability to solve for an unknown addend in a decimal addition equation, applying their understanding of place value, the structure of equations and inverse operations. In both parts, students are presented with a sum where one component is missing:

- In a.), they must determine what must be added to 1.8 to get 5.3.
- In b.), they must determine what must be added to 0.51 to get 2.

This requires either subtractive reasoning (e.g., 5.3 – 1.8) or conceptual understanding of the additive relationship. The key skill here is the ability to flexibly apply basic algebraic structure and decimal arithmetic.

Why is this skill a key skill?

Solving for unknowns in numerical equalities is a foundational key skill in bridging arithmetic and algebra. Within the DiToM framework, such tasks are seen as early algebraic reasoning—students must treat the equation as a whole and understand the structural role of the unknown. Furthermore, dealing with decimal values strengthens students fluency with base-ten structure and supports later success in topics like measurement, finance and proportional reasoning. The ability to move between knowns and unknowns using inverse operations reflects deeper operational understanding, recognizing equality sign as an equivalence relation and contributes to the development of equation sense.

What kind of errors and other warning signals can be expected with this task?

Some students may attempt to guess rather than apply subtraction, especially if they are unsure how to handle decimal place value. A typical error is misalignment of decimal digits (e.g., treating 1.8 as 18 or forgetting to align the tenths). In part B, students may confuse the position of the unknown and subtract 0.51 from 0 instead of from 2. Others may solve the equation by adding instead of subtracting, or may write a logically incorrect result that numerically “fits” but doesn’t respect decimal structure. These mistakes point to procedural gaps, insecurity with decimals, or a lack of equation interpretation skills.

What kind of support could be given to children who show deficits with this task?

Effective support includes practice in solving open number sentences using decimal number lines, bar models, or equation balance models to visualize relationships aswell as apply intelligent calculation strategies. Students should be encouraged to rewrite equations using subtraction to isolate the unknown and to estimate their result first to develop a sense of plausibility. Exercises that focus on aligning decimals and on decomposing numbers can help build fluency with decimal arithmetic. Encouraging verbal explanations (“What would you add to 1.8 to get 5.3?”) strengthens reasoning and connects arithmetic with algebraic structure.

Task 3.7: Performing decimal subtraction and multiplication

Key skill tested with this task

This task assesses students ability to accurately carry out arithmetic operations with decimal numbers. In part A, they are required to compute the difference between 23.5 and 1.12. In part B, they must determine the product of 6 and 2.5. Both problems test place value understanding, operational fluency and precision in working with decimal numbers—both in alignment and in calculation. The task reflects routine yet essential skills in the context of the decimal system.

Calculate:

a) $23{,}5 - 1{,}12 =$ _____

b) $6 \cdot 2{,}5 =$ _____

Why is this skill a key skill?

Performing basic operations with decimals is a foundational mathematical key skill, as defined in the DiToM framework. It is not only essential for everyday numeracy (e.g., dealing with money, measurements, or data), but also supports algebraic generalization and proportional reasoning. Accurate decimal computation underpins many mathematical domains, including geometry, statistics and problem solving in scientific contexts. The ability to compute with decimals reflects a deep integration of place value knowledge, algorithmic control and estimation strategies.

What kind of errors and other warning signals can be expected with this task?

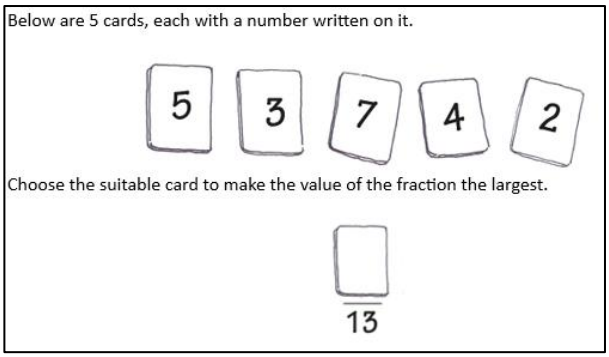
Common mistakes in part A include misalignment of decimal places (e.g., treating 23.5 as 23.50 but not aligning properly with 1.12), leading to incorrect subtraction. Students might also subtract digits from the wrong positions, or ignore the decimal altogether. Some may also use inappropriate strategies like repeated addition without structural control. These errors often indicate weak understanding of place value, operation properties, or decimal positioning.

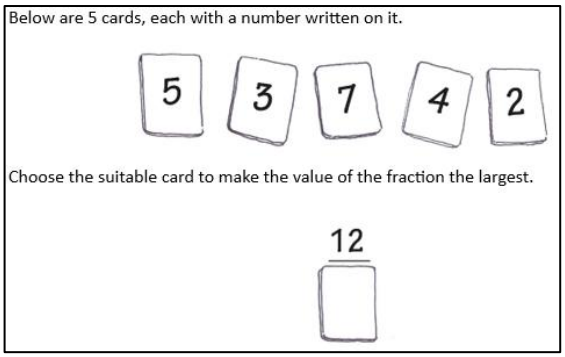
What kind of support could be given to children who show deficits with this task?

Support should include structured practice in place value alignment, especially with subtraction across decimal places. Teachers can use grid paper or place value charts to help students align digits correctly. Estimation strategies ("Is the answer going to be around 22 or 10?") help develop number sense and a feel for plausibility. For multiplication, using area models or base-ten block representations can support conceptual understanding of multiplying by decimals. It's also useful to model both standard algorithms and mental math strategies (e.g., $6 \cdot 2.5 = 6 \cdot 2 + 6 \cdot 0.5 = 15$ or $6 \cdot 2{,}5 = (3 \cdot 2) \cdot 2{,}5 = 3 \cdot (2 \cdot 2{,}5) = 3 \cdot 5 = 15$ (associativity of multiplication)). Frequent verbal reasoning strengthens understanding of how decimals behave in operations.

Task 3.8 and 3.9.: Maximizing the value of a fraction by choosing a suitable numerator or denominator

Key skill tested with this task





This task assesses students ability to reason about the structure of fractions and to apply their understanding to maximize a fraction's value by selecting the most appropriate number from a set of options. Students are presented with a number of cards, each showing a different number and are asked to insert one of them into a given fractional structure in such a way that the value of the resulting fraction is as large as possible. The fraction is typically either missing the numerator or the denominator and students must choose the number that will make the fraction's value greatest. This tests students flexible reasoning with ratios and relative magnitude.

Why is this skill a key skill?

Understanding how numerators and denominators affect the size of a fraction is a core idea in fraction learning. Within the DiToM framework, this ability reflects a deeper conceptual grasp of fraction magnitude—how increasing the numerator or decreasing the denominator impacts the overall value. It also supports the development of relational thinking, in which students think beyond surface features and instead reason structurally about number relationships. These insights are essential for later work in ratio, proportion, scaling and algebraic reasoning.

What kind of errors and other warning signals can be expected with this task?

A common mistake is selecting the largest number available, regardless of whether it is placed in the numerator or denominator, under the false assumption that "bigger is better." This indicates a procedural or surface-level strategy without structural understanding. Others may simply choose randomly or confuse the roles of numerator and denominator, for instance, maximizing the number itself rather than the value of the resulting fraction. These errors suggest fragile understanding of how fractions behave and limited experience with comparing non-unit fractions. Students may also misunderstand the goal (e.g., try to get closest to 1 rather than maximize value).

What kind of support could be given to children who show deficits with this task?

Effective support includes hands-on activities with fraction strips or number lines, where students manipulate numerators and denominators to observe how the size of the fraction changes. Teachers can model comparisons like "3/4 vs. 3/5" or "4/7 vs. 5/7" to explore how the numerator or denominator impacts magnitude. Discussion-based tasks—"Which is bigger and why?"—promote deeper reasoning. Visualizing fractions on a common number line or using software that dynamically shows fraction size can also help students grasp these relationships. Over time, guiding students toward generalizations ("A smaller denominator makes a fraction larger if the numerator is fixed") supports transfer and abstraction.

VI. Scientific evaluation

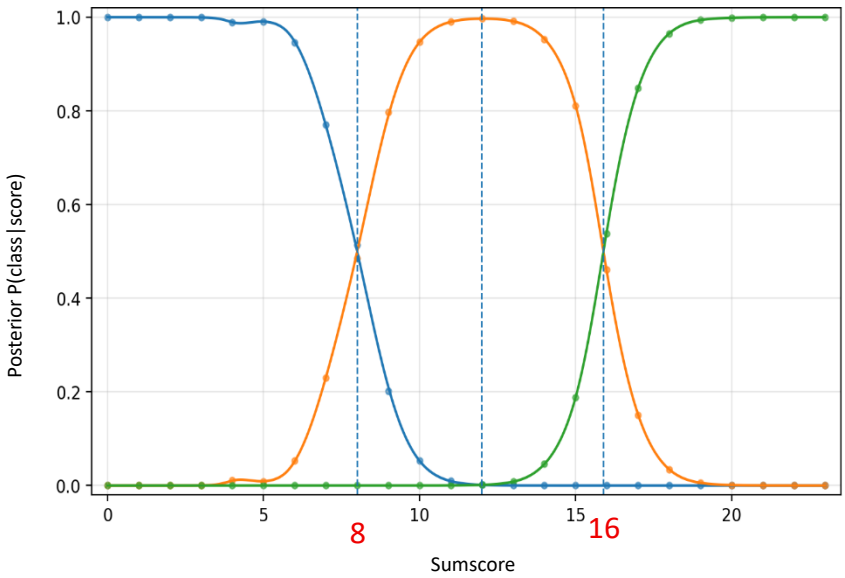
This DiToM Screening 6+ was developed on the basis of theory and tested as part of a non-representative validation study. The results reported below serve to identify students who are potentially at risk due to a lack of mathematical key skills for subsequent mathematics learning at school. The test helps teachers at the end of grade 6 / beginning of grade 7, to make an empirically based assessment of students' performance and to identify these with conspicuous results for appropriate support at an early stage.

Description of the sample and central results

The test was trialled from June to July 2025 within the last 3 weeks of the 2024/2025 school year with 1841 pupils from schools in Greece, Germany, France, Spain, Italy, Croatia and Sweden.

The test comprises the following test parts: Basic Arithmetic Skills with 8 items, Proportionality with 3 items, Technical Calculations with 12 items. If an item was solved correctly, 1 point was awarded; if the solution was incorrect, incomplete or missing, 0 points were awarded. The test was carried out according to standardised criteria (see IV. Implementation of the DiToM test) and evaluated. As the test is designed as a screening test that identifies students who are potentially at-risk, strong ceiling effects were expected (i.e. no normal distribution, but rather a left-skewed distribution) and desired. This was confirmed by the trial.

For practice-oriented communication, not a single threshold value is specified, but two threshold values that differentiate between potentially at risk, students who continue to be observed and students potentially not at-risk. The determination of the cut-off score was data-driven by a latent class analysis with 3 clearly distinguishable classes. The classes are non-overlapping and monotonic. The posterior probabilities of the class assignment were plotted against the point score, smoothed and used to determine the critical point thresholds for funding decisions with regard to their intersection points (see Figure 1). The intersection points of the curves were used (posterior probability p=0.5).



The class analysis shows three clearly distinguishable classes, which are interpreted as K1→ students with weak performance in the screening test, K2→ students with rather weak performance in the screening test and K3→ students with inconspicuous performance in the screening test.
To determine the critical score, the threshold up to which there is a 50% probability of being in the class with poor test performance was selected. This first threshold is therefore 8 points. Students who have achieved a **score of 8** points or less need support to work through the basics in order to be able to build up the following content of the maths lessons in an understanding-orientated way. The second threshold is 16 points. Students who have achieved a **score between 9 and 16 points** should be observed in maths lessons over the next few weeks to see whether they understand the content covered and can implement it independently.

VII. Evaluation sheet

The following scale provides initial indications of the skills with which students most probable score points in the following three ranges: 0-8 points, 9-16 points, and 17-23 points.

Score

23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0

This group is unremarkable in terms of their prerequisites for learning mathematics at school.

C: In addition to A and B, students are often able to…

… choose the correct fraction for a shaded circle, multiply natural numbers, symbolically represent decimal numbers/fractions on a number line, compare decimal numbers, maximize the value of a fraction by choosing a suitable denominator and perform decimal subtraction

And students are rarely able to…

… perform division with natural numbers and symbolically represent decimal numbers/fractions on a number line in more complex placements or mixed scales

This group should be monitored more closely in the coming weeks to see whether the following content can be understood and implemented independently.

B: In addition to A, students are often able to…

… perform division with natural numbers usually correct, identify and extend number patterns, find missing addends in decimal equations, shade a given fraction of a rectangle and multiply natural numbers

And students are rarely able to…

… perform simple decimal multiplication and equate quantities

This group is at risk in terms of their prerequisites for learning mathematics at school and should receive special support as quickly as possible.

A: Students are often able to…

… represent equivalent fractions, maximize the value of a fraction by choosing a suitable numerator, read decimal numbers from a thermometer, apply order of operations (priority rules), compare an improper fraction with natural numbers and interpret equivalent fractions

And students are rarely able to…

… use proportional reasoning with quantities and prices and translate written text into mathematical expressions

References:

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio and proportion. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296–333). New York: Macmillan.

Brings, L., & Kleine, M. (2025). Diagnostic Tool in Mathematics (DiToM): Development and evaluation of a screening instrument for early identification of at-risk students in lower secondary mathematics education. In *Proceedings of EDULEARN25 Conference*. Palma, Spain. Facebook+7DiToM - Diagnostic Tool in Mathematics+7iated.org+7

Ehlert, A., & Fritz, A. (2013). Arithmetische Basiskompetenzen von Schülerinnen und Schülern in den Klassen 5 bis 7 der Sekundarstufe. *Journal für Mathematik-Didaktik*, 34(2), 237–263.

Gaidoschik, M. (2025). *Das dezimale Stellenwertsystem: Verstehen, verinnerlichen und flexibel anwenden*. Hannover: Klett Kallmeyer.

Polotskaia, E., & Savard, A. (2021). Some multiplicative structures in elementary education: a view from relational paradigm. Educational Studies in Mathematics, 106(3), 447–469.

Prediger, S. (2008). Zahlaspekte verstehen und flexibel nutzen. In E. Cohors-Fresenborg et al. (Eds.), *Mathematiklernen ermöglichen* (pp. 85–100). Münster: Waxmann. ph-gmuend.de+3Edoc LMU München+3Wikipedia+3

Radford, L. (2014). The progressive development of early embodied algebraic thinking. Mathematics Education Research Journal, 26(2), 257–277. doi.org/10.1007/s13394-013-0087-2

Siegler, R. S., & Booth, J. L. (2005). Development of numerical estimation in young children. *Child Development*, 76(2), 428–444.

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, 107(3), 909–918.

Treppo, A., & van den Heuvel-Panhuizen, M (2014). Visual representations as objects of analysis: the number line as an example. ZDM, 46, 45–58.

Van Dooren, W., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication... and back: The development of students additive and multiplicative reasoning skills. *Cognition and Instruction*, 28(3), 360–381.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 557–628). Charlotte, NC: Information Age.

Wittmann, E. Ch., & Müller, G. N. (2004). *Handbuch produktiver Rechenübungen*. Seelze: Friedrich Verlag.