



# Screening 4+

## Handbook for teachers



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## Foreword

This manual is designed to support you in administering the *DiToM 4+* screening and in using the test results effectively with your class. On the following pages, you will find:

a brief introduction to the aims and guiding principles of the Erasmus+ project *DiToM*;

detailed, step-by-step instructions for conducting the *DiToM 4+* screening in the classroom;

concise explanations of each task in the *DiToM 4+* screening, including notes on possible support strategies for children whose screening results indicate learning gaps in key mathematical competencies;

guidance on how to evaluate and document the results.

The administration guide (Section 2) and the evaluation tables (Section 4) can also be downloaded separately as individual PDF files from [www.ditom.org/](http://www.ditom.org/)

We recommend printing the administration guide double-sided and spiral-binding it. In the booklet that you will get, you can keep the teacher-facing page for reading the instructions aloud, while the page facing the children often includes an example that helps you explain what the children are expected to do.

# 1) Aims and guiding principles of DiToM

Mathematics learning progresses in stages: new knowledge builds on secure prior understanding. When fundamental ideas and concepts are missing, students find it increasingly difficult to grasp and make sense of mathematical content that builds upon that foundation. Both national and international studies show that a significant proportion of students already fail to meet the minimum standards in mathematics at the primary level - and, for the reasons described above, almost inevitably continue to struggle in secondary school. Alarming, many young people finish their compulsory education without having achieved the basic level of mathematical literacy that, according to the OECD, is essential for “full participation in social life.”

To counteract this, teachers must first be able to identify mathematical learning difficulties - ideally early and as precisely as possible. Only on this basis can targeted support measures be taken. This is precisely where the EU project *Diagnostic Tools in Mathematics (DiToM)* comes in. In a partnership between Germany, France, Greece, Croatia, Italy, Sweden, and Spain, five interconnected screening instruments were developed. These tools enable teachers, at the end or beginning of a school year, to gain a concise overview of which students are at risk of falling behind in mathematics if they do not receive targeted support measures.

The screenings follow a two-year cycle:

- **Screening 0** – Beginning of primary school
- **Screening 2+** – End of Grade 2 / Beginning of Grade 3
- **Screening 4+** – End of Grade 4 / Beginning of Grade 5
- **Screening 6+** – End of Grade 6 / Beginning of Grade 7
- **Screening 8+** – End of Grade 8 / Beginning of Grade 9

## What are the *DiToM* Screenings and what do they achieve?

The five screenings are paper-and-pencil tests focused on key mathematical competencies that should be secure at the start of a grade level for new content to be learned with understanding. Each test can be administered to the whole class within a single lesson and, using the provided scoring

tools (see Section 4), evaluated with relatively little time investment. The results give teachers an initial structured overview of which children are likely to need additional support in particular areas. The word “*likely*” is crucial: a screening does **not** replace an individual, qualitative assessment of a child’s learning status. At best, it provides initial clues as to what strategies or solution approaches a child may have used. More detailed understanding requires targeted observation and one-on-one discussions, using more finely differentiated tasks. The screening, however, can serve as a valuable starting point to determine which children would benefit most from such follow-up assessments.

### What are “key mathematical competencies”?

As noted earlier, school mathematics is characterized by an “*internal learning hierarchy*” (Wittmann, 2015, p. 199). This is particularly true in the domains of arithmetic (numbers and operations) and algebra - the very areas that *DiToM* screenings intentionally focus on. In these domains, it is possible at every learning stage to identify *key competencies*—those without which further learning cannot take place meaningfully and sustainably.

For example: To work successfully with natural numbers, children must understand them in terms of the *part–whole concept*—a developmental process that should be completed during the first school year. The part–whole concept means, for instance, that the number seven is understood as a whole composed of parts - five and two, four and three, one and six, and so on. This understanding should then become automatic: a child should no longer need conscious effort to recognize five as the missing part of seven when two is given as the other part. In other words, children should automatically think of numbers in terms of their decompositions and relationships. This combination of *understanding* and *automation* is characteristic of many key competencies: only once certain skills are automatic can mental capacity be freed up to tackle higher-level mathematical challenges.

Whether the key competency of “thinking of numbers as compositions” (or “number decomposition”) is well established can be seen, for instance, in a child’s calculation strategies. A child who thinks of seven as five and two will solve  $7 - 5$  effortlessly, even in the first school year, without counting. Children who lack this competency, however, often continue to rely on laborious, error-prone counting strategies well into later primary and secondary grades. Counting-based addition and subtraction soon become unmanageable when two- or three-digit numbers are involved. Such children also struggle to use relationships between multiplication facts - for instance, recognizing that  $9 \times 6$  is six less than the easily remembered  $10 \times 6$ . Deficits in one key competency (understanding

numbers as compositions) thus hinder the acquisition of others (addition, subtraction, multiplication), which in turn are prerequisites for more advanced skills (division, proportional reasoning, etc.).

This chain continues beyond primary school: students who struggle with natural numbers will face even greater difficulties with fractions and decimals. Algebra, later on, builds on insights that should have been gained from working with the basic operations in primary school. Without those insights, algebra can appear to students as an indecipherable code.

For this reason, the *DiToM* screenings focus on key competencies - those that should be securely established at the start of Grades 1, 3, 5, 7, and 9, so that further mathematical learning can proceed successfully.

### After administering the *DiToM* screening – what’s next?

Using the evaluation tools described in Section 4, teachers create a table (Excel or paper) that can be read in two directions:

- **Across rows:** Each child’s results show which tasks were solved correctly, partially correctly, incorrectly, or left blank—resulting in an overall score for that child.
- **Down columns:** For each task, the table shows how many children solved it correctly, partially correctly, incorrectly, or not at all.

#### **With a view on individual students:**

*DiToM* is not about labeling children. The screenings are not designed to identify students with “dyscalculia.” Clinical diagnoses of that kind do not address the core question that *DiToM* seeks to answer: *How can teachers best support children struggling with key arithmetic competencies?* Targeted support requires an accurate understanding of each child’s current learning level. *DiToM* helps identify those for whom such detailed assessment is urgently needed—nothing more, but also nothing less. Section 3 provides brief notes on what kinds of follow-up support may be helpful for each specific task.

The “critical threshold scores” given in Section 4 were determined based on trials of the *DiToM* screenings with 8,820 children across the seven partner countries. Using *latent class analysis* (see Livingston, 2014), children were grouped as follows:

- Group A: Children showing widespread difficulties across several key competencies.
- Group B: Children showing indications of difficulties in specific areas.
- Group C: Children showing no major indications of difficulty.

It is important to remember that any screening captures only a *snapshot*. Some children may simply have had a bad day or been distracted, others might have—despite precautions—copied answers. Screening results should therefore be interpreted cautiously. They should always be compared with observations from daily classroom, and used as a prompt for further targeted observation and follow-up tasks in the coming days and weeks.

If it becomes clear that a child falls into **Group A**, there is reason to expect that their mathematical difficulties will worsen over the school year unless timely and effective interventions are implemented. Section 3 can only suggest general directions for such interventions, based on the key competencies assessed by each task. For more extensive guidance, teachers must refer to the relevant educational literature.

Children in **Group B** are also likely to need targeted support in at least some areas to progress successfully in their learning. It is worth remembering that all screening tasks assess *key* competencies. The screening is intentionally designed *not* to distinguish among high achievers—ideally, most children should find the tasks quite easy. Therefore, any errors made by **Group C** children on individual tasks should also be taken seriously, as they may reveal gaps in key foundational skills.

**With a view on the class as a whole:**

The latter applies particularly when the results show that multiple children struggled with the same task. This may indicate that they have received insufficient or unfocused practice with that competency, either in their prior schooling or before entering school. In such cases, it is all the more important that these learning opportunities now be provided, even if the curriculum has already moved on to new content. Again, it is important to take into account the hierarchical structure of mathematics learning: each level depends on secure understanding of the foundational competencies before moving forward.

## 2) Instructions for administering Screening 4+

The *DiToM* 4+ screening is designed for use with the entire class at the end of Grade 4 or immediately at the beginning of Grade 5. It covers the following content areas:

- 1 Writing numbers
- 2 Comparing numbers
- 3a Adding  $\frac{1}{10}/\frac{1}{100}$  together with bundling
- 3b Taking away  $\frac{1}{10}/\frac{1}{100}$  together with unbundling
- 4 Numbers on the number line
- 5 Halving numbers up to 10,000
- 6a Mental calculation: Addition and subtraction
- 6b Mental calculation: Dealing with zeroes
- 7a Written addition
- 7b Written subtraction
- 8 Operational understanding of addition/subtraction
- 9 Basic number facts – multiplication
- 10 Basic number facts – division
- 11 Mental calculation: Dealing with zeroes
- 12 Operational understanding: Representations
- 13 Operational understanding: Word problems

The following section provides detailed, task-by-task instructions on what to tell the children before and during the administration of the test.

These instructions are also available as a **separate PDF file for download**. If you print this file double-sided and bind it with a spiral, you will have a booklet from which you can read the instructions aloud during the test and refer back to key points to keep in mind during administration,

## Before and during the distribution of test booklets

- Explain to the children that at the end of Grade 4 / beginning of Grade 5 you would like to find out what they already know and can do.
- Tell them that each child will now receive a booklet with tasks to complete one after another.
- Emphasize that it is important for each child to work independently and that copying from a neighbour is not helpful. Another child's answer might be wrong — and most importantly, you want to know what each child can already do well on their own and where they might still need help.
- If necessary and possible, place school bags (or similar items) between children during the test to make copying more difficult.
- Ask the children to write with a pencil. Since erasing takes time, they should simply cross out any mistakes and write the correct answer next to them. You may wish to demonstrate this briefly on the board.
- You will lead them through the tasks one by one and explain what to do for each task. It is important that they pay attention and listen carefully to your instructions.
- Tell the children that the tasks are to be done one after another, and that you will always explain what to do before they start. Sometimes there will also be an example. Remind them not to continue on their own, even if they finish a task earlier than others. They should only turn the page when you tell them to do so.
- Make sure all desks are empty and that each child has only one sharpened pencil in front of them.
- Some tasks have a **time limit**. To avoid stress, do not announce this in advance. Instead, tell them that you expect that they will solve some of the tasks rather quickly because they probably already know them by heart. Announce that when they have worked for some time on a task, you may say STOP, and then everyone should indeed stop writing. Emphasize that it is not a problem if someone has not finished at that point of time. For all the screening, the goal is a calm, stress-free environment.

- For **tasks without a time limit**, use your own judgement about when to say STOP. This may be advisable for some tasks, once most children have finished. Some children may take considerably longer than the big majority, and even with more time, might not complete the task. However, if others have to wait too long, restlessness may arise. Therefore, it might be better to say STOP and assure those who have not finished that it does not matter, and praise the children for their efforts.
- **Now hand out the booklets.** Emphasize that they must remain closed on the desks until you tell the children to turn to the first task. Ask them first to write their name on the cover page.

# 1 Writing numbers

No example for this task

## Screening task

a) _____
b) _____
c) _____

No time limit!

“Now please turn your page over to the first task.”

„You see three empty lines a) to c).”

„I will dictate three numbers to write down on the lines one below the other.”

„These are the three numbers:”

→ *Read each number twice!*

*After the first/second number, say: „Now listen to the next number.”*

a) **five thousand and eighty-nine** (5,089)

b) **forty-three thousand and five** (43,005)

c) **three-hundred thousand five-hundred** (300,500)

„Now let us look at the next task. Do not yet turn the page!”

## 2 Comparing numbers

### Example

→ write the following examples on the board

500 550

600 550

“Let us compare the first pair of numbers: 500 is *smaller* than 550.

So, we write the sign for *smaller than* in between:  $500 < 550$ ”

→ write the  $<$  symbol between the first number pair

„Now let’s look at the second pair of numbers. 600 is bigger than 550.

So, we write the sign for *bigger than* in between:  $600 > 550$ ”

→ write the  $>$  symbol between the second number pair

### Screening task

No time limit!

“Now please turn your page over to the next task.

Here you see three other number pairs. Compare the numbers and write the correct symbol in between.

a) 6,001          5,999

b) 7,955          7,599

c) 99,899        102,101

Once you are finished, put your pen down.

Now I explain to you the next task. Do NOT yet turn the page!”

Editorial note: Check if the four-digit numbers are written with the dot or space in your country!

### 3a Adding 1/10/100 together with bundling

#### Example

“The next task is about what is more than a given number.  
I give you an example:”

→ write 1 more than 236: \_\_\_\_\_ on the board

„One more than 236 is ... (let the students answer first) .. 237.”

→ write 237 on the line beside of 236

Next example:

→ write 10 more than 350: \_\_\_\_\_ on the board

„Ten more than 350 is ... (let the students answer first) ... 360.”

→ write 360 on the line beside of 350

Last example:

→ write 100 more than 570: \_\_\_\_\_ on the board

„Hundred more than 570 is ... (let the students answer first) ... 670.”

→ write 670 on the line beside of 570

#### Screening task

No time limit!

“Now please turn your page over to the task.

Here you see three numbers.

Your task is to find out what is 1 more, then 10 more, then 100 more.

Think carefully and write the correct numbers on the lines.

- |    |                            |
|----|----------------------------|
| a) | 1 more than 9,899: _____   |
| b) | 10 more than 4,590: _____  |
| c) | 100 more than 3,900: _____ |

Once you are finished, put your pen down.

Now I explain to you the next task. Do NOT yet turn the page!”

### 3b Take away 1/10/100 together with unbundling

#### Example

“This next one is similar to the one you just made.

But this time it is always about what is less then the given number.”

Here is an example:

→ write 1 less than 236: \_\_\_\_\_ on the board  
**„One less than 236 is ... (let the students answer first) .. 235.”**

→ write 235 on the line beside of 236

Next example:

→ write 10 less than 350: \_\_\_\_\_ on the board  
**„Ten less than 350 is ... (let the students answer first) ... 340.”**

→ write 340 on the line beside of 350

Last example:

→ write 100 less than 570: \_\_\_\_\_ on the board  
**„Hundred less than 570 is ... (let the students answer first) ... 470.”**

→ write 470 on the line beside of 570

#### Screening task

No time limit!

“Now please turn your page over to the next task.

Here you see three numbers.

You have to find out what is 1 less, then 10 less, then 100 less.

Write the correct numbers on the lines.

- |    |                            |
|----|----------------------------|
| a) | 1 less than 7,000: _____   |
| b) | 10 less than 3,500: _____  |
| c) | 100 less than 4,000: _____ |

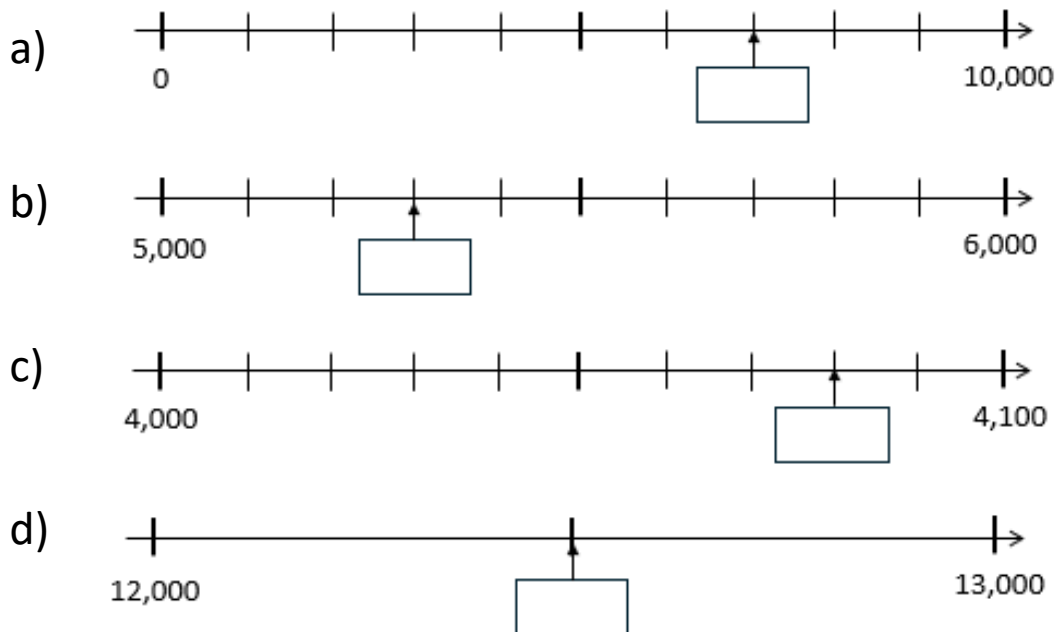
Once you are finished, put your pen down. Now we go to the next task. This time, we do not need an example. Please turn over to the next page.”

## 4 Numbers on the number line

No example needed

No time limit!

### Screening task



“Here you see four different number lines.

Write the missing numbers in the boxes.

The arrow points to the number you are looking for.

**But look carefully! The number lines are all different.**

On each number line, mind the numbers that are already written and how many marks there are between these numbers!

Once you are finished, put down your pen please.

Do not turn the page yet. First, I will explain the next task to you.”

## 5 Halving numbers up to 10,000

### Example

„The next task is about halving.

Here is an example”:

→ write half of 400: \_\_\_\_ on the board

„Half of 400 is .... (let the students answer first) .. 200.”

→ write half of 400: 200 on the board

### Screening task

a) Half of 1,000: \_\_\_\_\_

b) Half of 500: \_\_\_\_\_

c) Half of 700: \_\_\_\_\_

d) Half of 3,000: \_\_\_\_\_

Time limit: 40 sec

“Now please turn your page over to the new task.”

„You see four more numbers.

Write down the half of these four numbers.

Start now!”

→ Count to 40 in your head!

„Now lay down your pencil. It doesn't matter, if you have not yet finished. Please do not write anymore on this page but listen to me.

I will explain the next task to you. Do NOT yet turn the page.”

## 6a Mental calculation: Addition and subtraction

No example needed

### Screening task

a)  $248 + 52 =$  \_\_\_\_\_

b)  $637 + 99 =$  \_\_\_\_\_

c)  $723 - 24 =$  \_\_\_\_\_

d)  $453 - 99 =$  \_\_\_\_\_

Time limit: 60 sec

“Our next task is about adding and subtracting.

On the next page, you will see two addition and two subtraction tasks.”

„Have a good look at the **numbers, before you start calculating.**

These are special numbers, try to find an **easy** way to calculate.”

„Calculate in your head and just write down the results.”

**„Now turn the page to the next task.**

As said: Look at the numbers, before you calculate, and be careful:

First there are two addition tasks, then two subtraction tasks.

Start now.”

→ *Count to 60 in your head!*

„Please lay down your pencil now.

Once more, it doesn't matter, if you have not yet finished!

Please do not write anymore on this page but listen to me.”

„I will explain the next task to you.”<sup>17</sup>

## 6b Mental calculation: Dealing with zeroes

No example needed

### Screening task

a)  $3,600 + 900 =$  \_\_\_\_\_

b)  $56,000 + 8,000 =$  \_\_\_\_\_

c)  $3,200 - 700 =$  \_\_\_\_\_

d)  $54,000 - 5,000 =$  \_\_\_\_\_

Time limit: 60 sec

“On the next page, you will find some more tasks.”

„Turn the page now to the next task.”

„Again: Take a good look at the numbers,  
and pay attention to the plus and the minus sign.”

„Start now.”

→ *Count to 60 in your head!*

„Please lay down your pencil now.

Once more, it doesn't matter, if you have not yet finished!”

„Please turn over to the next page.”

## 7a Written addition

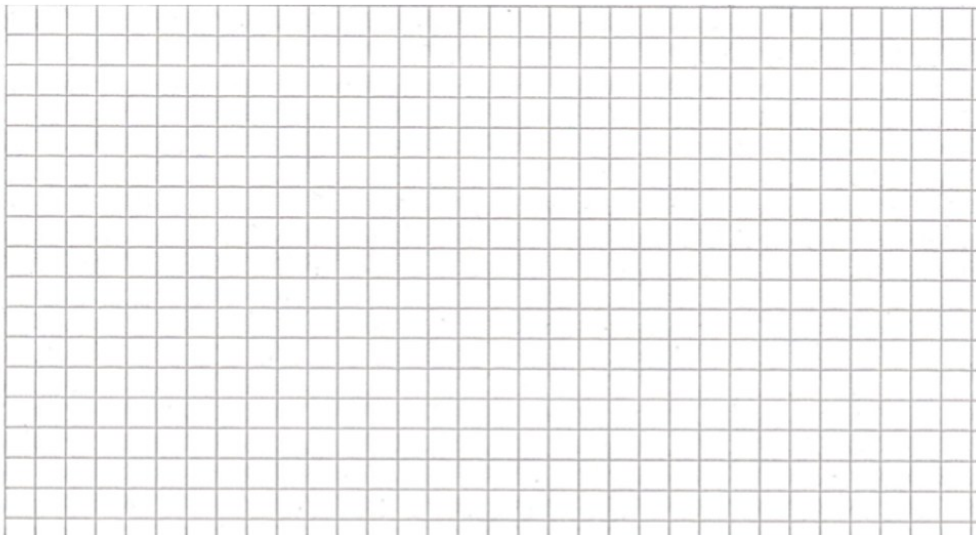
No example needed

Screening task

No time limit!

a)  $548 + 36$

b)  $760 + 564$

A large grid of graph paper, consisting of 20 columns and 20 rows of small squares, intended for students to perform the addition tasks.

“Here you find two more addition tasks.”

„This time, they should be done using **long** addition.”

„Begin each task by writing the two numbers one below the other and then do the calculation.”

„Once you are finished, put down your pen please.

Do not yet turn over to the next page.”

## 7b Written subtraction

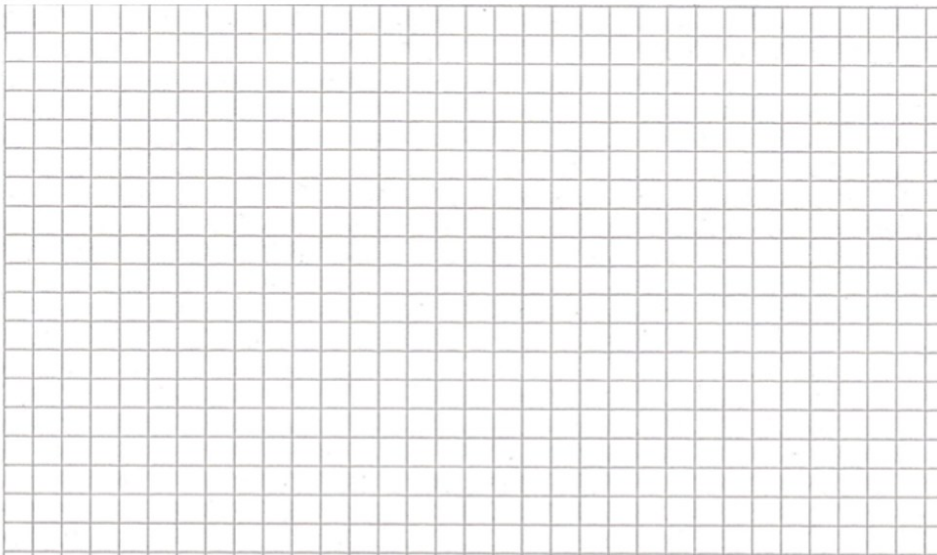
No example needed

Screening task

No time limit!

a)  $711 - 67$

b)  $806 - 534$



“Now please turn your page over. You find two more tasks, this time it is about subtraction.”

„The tasks should be done using **long** subtraction.”

„Begin each task by writing the two numbers one below the other and then do the calculation.”

„Once you are finished, put down your pen please.

Do not yet turn to the next page but wait until I tell you to do so.”

## 8 Operational understanding of addition/subtraction

No example needed

### Screening task

David is 35 years old.  
He is 4 years older than Helen.  
How old is Helen?  
Calculation: \_\_\_\_\_  
Answer: Helen is \_\_\_\_\_ years old

No time limit!

“Now please turn your page over.

The next task is a word problem. I read it to you:”

„David is 35 years old. He is 4 years older than Helen.

How old is Helen?”

→ **Read it out TWICE!**

„Write down your calculation and your answer.

**It is not enough to write only the result, you must also show the calculation!”**

„Once you are finished, please put your pencil down and wait.”

„Before you turn the page, I explain the next task.

It is about the multiplication.”

„Try to write down the results as quick as you can but also try to have all results correct!”

## 9 Basic number facts multiplication

No example needed

### Screening task

a)  $6 \times 1 =$  \_\_\_\_\_

b)  $10 \times 8 =$  \_\_\_\_\_

c)  $8 \times 4 =$  \_\_\_\_\_

d)  $7 \times 9 =$  \_\_\_\_\_

e)  $9 \times 0 =$  \_\_\_\_\_

f)  $7 \times 5 =$  \_\_\_\_\_

Time limit: 30 sec

“Now you can turn the page.”

„Here are the tasks you should solve. Start now!”

→ *Count to 30 in your head!*

„Please lay down your pencil now.

Don't stress yourselves, it doesn't matter, if you have not yet finished all the tasks!„

„Do not write anymore on this page, I explain the next task.”

„On the next page, you will find some division tasks.”

„Again, try to write down the results as quick as you can but also try to have all results correct!

And no stress – just do it the best you can!”

„Turn the page to the next task now!”

## 10 Basic number facts division

No example needed

### Screening task

a)  $80 : 10 =$  \_\_\_\_\_

b)  $6 : 6 =$  \_\_\_\_\_

c)  $28 : 4 =$  \_\_\_\_\_

d)  $72 : 9 =$  \_\_\_\_\_

e)  $30 : 5 =$  \_\_\_\_\_

f)  $7 : 1 =$  \_\_\_\_\_

Time limit: 30 sec

“Here are the division tasks. Start now!”

→ *Count to 30 in your head!*

„Please lay down your pencil now..”

„Once more: It is absolutely no problem if you have not yet finished all the tasks!”

„Please turn over to the next page!”

## 11 Mental calculation: Dealing with zeroes

No example needed

### Screening task

a)  $7 \times 5,000 =$  \_\_\_\_\_

b)  $50 \times 20 =$  \_\_\_\_\_

c)  $60,000 : 100 =$  \_\_\_\_\_

d)  $3,000 : 5 =$  \_\_\_\_\_

No time limit!

“Here you see two multiplication and two division tasks.

This time, the numbers are big, so it is OK if you take some more time.

**Pay attention to the zeros!”**

„Do the tasks in your head and write down only the result.

Be careful: First, there are two multiplications, but then there are two division tasks!”

„Start now. Once you are finished, please put you pencil down and wait.”

„Well done! Now you really did a lot of computing!”

„We will move on. There are only a few more tasks to do,  
and no more calculations to compute.”

„Please turn over to the next page.”

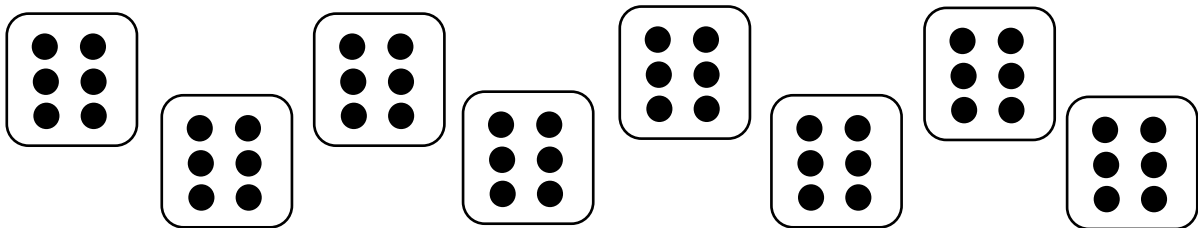
## 12 Operational understanding: Representations

No example needed

No time limit!

### Screening task

There is a calculation to figure out the total number of dots below.



Write down a multiplication task that fits the picture!

You do not need to write the resulting total number of dots!

Calculation : \_\_\_\_\_

“Look at this picture. Here you can see 8 dices that are equal!

To figure out the total number of dots, you could count all the dots, but this is rather tedious. But it is also possible to find this total number of dots with a calculation.”

„Your task is to write down a **multiplication task** that fits the picture!

You do not have to write the result, but just the calculation that leads to the total number of dots shown in the picture!”

„Write down the calculation on the line!”

„Once you are finished, put down your pen please, and wait.”

...

„Now turn over to the next page. It is our last task.

Let me first explain it to you.”

### 13 Operational understanding: Word problems

No example needed

No time limit!

#### Screening task

a)	A baker buys 24 packets of eggs. In each packet there are 6 eggs. How many eggs does the baker buy?	$24 : 6$	1
b)	24 eggs are packed into packets. Each packet holds 6 eggs. How many packets get filled?	$24 - 6$	2
c)	There are 24 eggs in the fridge. The cook takes 6 eggs out of the fridge. How many eggs remain in the fridge?	$24 \times 6$	3
		$24 + 6$	4

“Here you see three different word problems and four different calculations, all with the same numbers.”

„I will first read out the three word problems shown here on the lefthand side.”

→ *Read them out one after the other!*

„On the righthand side you see four calculations.

Which calculation fits to each of the three word problems?

Draw a line between each word problems and the matching calculation. You do not have to calculate and write the result.

Just connect each problem with the fitting calculation.

Of course, one of the calculation does not fit to any of the problems!”

„Once you are finished, put down your pen please and close your booklet. I will come around and collect it.”

→ *After collecting all the booklets<sup>26</sup> thank the children for their hard work and award them with a game or ...*

### 3) Explanations and suggestions for support regarding the single tasks of *DiToM 4+*

#### Task 1: Writing numbers

##### Key skill tested with this task

This task assesses students' ability to translate what they hear into the correct written numeral, that is, placing the digits in the correct order.

a)	5,089
b)	<del>300,500</del> 43,005
c)	

##### Why is this skill a key skill?

Being able to hear a number and write it correctly in digits is a foundational numeracy skill because it shows students can:

- translate between spoken and written representations, which is crucial for following mathematical instructions, interpreting word problems, and communicating mathematical ideas clearly;
- work fluently with numbers, which means they are better prepared for arithmetic, comparing and ordering numbers, recognizing numerical patterns;
- use working memory and attention, which means students can hold the number in mind and sequence the digits correctly, strengthening overall thinking and memory skills;
- apply number knowledge in everyday situations, for example writing dates, measurements, and personal information.

##### What kind of errors and other warning signals can be expected with this task?

When students are asked to write numbers such as 5,089, 43,005, or 300,500, various types of errors can occur, each indicating different challenges in processing and recording numbers. They may misplace or reverse digits, for example writing 5,089 as 5,809, which shows difficulty maintaining the correct order of digits. Numbers containing zeros in the middle or between digits often present particular problems, as students may omit or misplace these zeros, for instance writing 43,005 as 4305, or 300,500 as 3,005,000. Some children may also struggle with the use of separators, either omitting them entirely or misinterpreting the comma or full stop as a decimal point. Additionally, they may skip parts of the number altogether, suggesting problems with working memory or sustaining attention while writing. Students might also mishear or misunderstand the number, confusing similar-sounding digits or requiring the number to be repeated several times, which can indicate difficulties with listening comprehension or focus. Another possible error is that students write the words in the order in which they hear them: five thousand eighty-nine as 5,000,809.

##### What kind of support could be given to children who show deficits with this task?

Children who struggle to write numbers that are read aloud often benefit from support that emphasizes recognizing patterns in numbers, rather than writing each digit individually. For numbers with more than four digits, it is helpful to show students that numbers consist of repeating groups of three digits – Hundreds, Tens, and Ones – and how these groups form thousands, ten-thousands, hundred-thousands, and beyond. Using visual tools such as charts, number grids, or place-value mats can make these patterns easier to understand. Practice should encourage students to write each group as a unit and connect the spoken number to these blocks, rather than focusing only on individual digits. Strengthening listening, memory, and sequencing skills remains useful, but the main aim is to help students recognize and use the repeating patterns in large numbers so they can write them accurately and confidently. Also, oral exercises in which teachers count aloud in increments of 10 or 100 while writing down the numbers can be a useful approach.

## Task 2: Comparing numbers

### Key skill tested with this task

This task assesses students' ability to compare multi-digit numbers, and to represent the comparison accurately using the symbols for greater than, less than, or equal to.

### Why is this skill a key skill?

Comparing numbers is a key skill because it is fundamental to understanding the size and order of numbers. When students compare numbers, they start to recognize patterns in digits and how numbers increase or decrease, which supports the development of place-value understanding. This skill also strengthens logical thinking and decision-making, as students must analyze numbers carefully to determine which is greater, smaller, or if they are equal. Comparing numbers forms the basis for many future mathematical tasks, including understanding inequalities, solving arithmetic problems and working with decimal numbers.

### What kind of errors and other warning signals can be expected with this task?

When students are asked to insert the correct symbol ( $<$ ,  $>$ ,  $=$ ) between numbers such as 5,999 and 6,001, then between 7,955 and 7,599, or between 99,899 and 102,101, several common errors may occur. A common mistake is focusing on the last digit rather than the whole number. For example, a student might write  $5,999 > 6,001$  because 9 is greater than 1 in the ones place. Similarly, in the example with 7,955 and 7,599, a learner might compare only the last two digits (55 vs. 99) and incorrectly conclude that  $7,955 < 7,599$ , ignoring the hundreds place. Another frequent error is digit-count confusion, where learners think that  $99,899 > 102,101$  because "99..." seems greater than "10...". Some may also misuse the equals sign, for example, writing  $7,955 = 7,599$  because the numbers look very similar. There is also the possibility of symbol orientation mistakes, where a student knows that 5,999 is smaller than 6,001 but still writes  $5,999 > 6,001$  showing confusion about the direction of the inequality sign. Warning signals to look for include students making comparisons slowly or with hesitation, consistently relying only on the last digit, repeatedly using "=" as a safe guess, or applying faulty shortcuts such as "the bigger last digit means the bigger number." All of these indicate misunderstandings about place value or the correct use of comparison symbols.

### What kind of support could be given to children who show deficits with this task?

When students have difficulty comparing numbers, we often observe two main types of errors. Some students know which number is larger or smaller but use the wrong symbol ( $<$ ,  $>$ ,  $=$ ), while others struggle to determine which number is larger, sometimes becoming confused by the face value of single digits. Support can be tailored to each situation. For students who mix up symbols, it is helpful to teach the meaning of each symbol, have them verbalize their reasoning before writing, and provide practice focused on correct symbol use. For students who struggle with understanding which number is larger, teachers can guide them to compare numbers digit by digit from left to right, start with simpler whole numbers before progressing to larger numbers and decimals, and encourage them to explain their thinking, so they develop genuine understanding rather than simply following rules.

a)	6,001	5,999
b)	7,955	7,599
c)	99,899	102,101

### Task 3a: Adding 1/10/100 together with bundling

#### Key skill tested with this task

This task assesses students' place-value understanding and their ability to add 1, 10, or 100 correctly, including handling bundling or carrying, and working accurately with multi-digit numbers.

1 more than 9,899: \_\_\_\_\_

10 more than 4,590: \_\_\_\_\_

100 more than 3,900: \_\_\_\_\_

#### Why is this skill a key skill?

Adding 1, 10, or 100 with bundling is a key skill because it helps students understand how numbers change based on place value. It reinforces the idea that digits in different positions represent different values and demonstrates how to carry or "bundle" when a digit exceeds 9. Mastering this skill is essential for mental calculation, addition with larger numbers, and later arithmetic operations, as it builds both conceptual understanding and procedural fluency. It also helps students recognize patterns in numbers and apply addition in practical, real-life situations.

#### What kind of errors and other warning signals can be expected with this task?

Children might add to the wrong place, for example adding 100 to 3,900 and writing 3,910 instead of 4,000, or adding 10 to 4,590 and writing 4,690. Students may fail to carry or bundle correctly, not realizing that adding 1 to 9,899 increases the tens and hundreds digits as needed to reach 9,900.

They might misunderstand what "1 more", "10 more" or "100 more" means, applying the addition to the wrong digit. Another common mistake is following simple patterns incorrectly, such as always increasing the last digit or only one place regardless of the number's structure. These errors often indicate difficulties with place-value understanding, carrying/bundling, and connecting addition to the correct position in multi-digit numbers, which are essential for accurate calculation and mental arithmetic.

#### What kind of support could be given to children who show deficits with this task?

If a child struggles with adding 1, 10, or 100, it is important first to assess their understanding of the decimal system and place value. Support can start with concrete activities using manipulatives, such as units, tens, and hundreds, to form bundles and help them see how numbers change when adding. Next, focus on tasks where carrying/bundling is required, for example adding 1 to 9.899 or 100 to 3.900. Gradually, children should practise solving these tasks without materials, building on their earlier concrete experiences. Throughout, encourage children to verbalize their reasoning, first with guidance and later independently, to support internalization of the bundling principle and accurate addition.

### Task 3b: Take away 1/10/100 together with unbundling

#### Key skill tested with this task

This task assesses students' place-value understanding and their ability to subtract 1, 10, or 100 correctly, including handling unbundling or borrowing, and working accurately with multi-digit numbers.

1 less than 7,000: \_\_\_\_\_

10 less than 3,500: \_\_\_\_\_

100 less than 4,000: \_\_\_\_\_

#### Why is this skill a key skill?

A sound understanding of the decimal place value system forms the basis for flexible calculation with multi-digit numbers (and later decimal numbers), and for relating these numbers to each other and to the world in which we live (e.g., estimating, making rough calculations, and correctly assessing quantitative proportions in real-life situations).

The ability to solve tasks involving the unbundling of decimal units indicates a higher level of understanding than tasks involving bundling; difficulties with unbundling are more common.

#### What kind of errors and other warning signals can be expected with this task?

Children might respond that 1 less than 7,000 is 6,000, or (if they have learned that such answers should include one or more 9s) 7,009 or 6,009 (other answers are possible). 10 less than 3,500 might be, among others, 3,400 or 2,500. 100 less than 4,000 might be 3,000 (other incorrect answers may also occur).

Apart from errors or non-responses, it should be considered a warning sign if a child relies on algorithmic subtraction to find answers, even if the answers are correct. Children should be able to state the answers quickly, based on their understanding that one thousand equals ten hundreds, one hundred equals ten tens, and so on.

#### What kind of support could be given to children who show deficits with this task?

As stated, place value understanding is multi-faceted. If a child has difficulties with this task, a more comprehensive assessment of their current understanding of the principles of the decimal system is recommended.

Depending on the results of such a detailed clarification of the learning level, it may be necessary to revisit a fundamental concept of the bundling principle with the child. It is important to use tasks in which the child forms decimal bundles with appropriate materials. On this basis, the focus should shift to specific tasks that require unbundling, such as in *DiToM*-Task 3b or 5 (for example, halving 3,000). As always, it is important to help children progress to solving such tasks without materials, drawing on their previous experiences with material actions. For effective internalisation, children should be repeatedly encouraged to verbalise their actions, both as they perform them and increasingly in anticipation.

## Task 4: Numbers on the number line

### Key skill tested with this task

This task assesses students' ability to recognize intervals and distances between numbers on a number line and to place numbers (up to 100 000) in the correct position on number lines.

### Why is this skill a key skill?

Placing numbers correctly on a number line is an aspect of understanding numerical relationships, particularly ordinal numbers (which numbers come before or after others). It helps students visualize the size and position of numbers, develop a strong sense of numerical magnitude, and understand relationships between numbers. It reinforces place value understanding by showing how digits contribute to the overall value of a number.

Representing numbers on a number line is essential for many other skills, such as rounding numbers and estimating sums and differences. The representation of numbers on a number line is later used for fractions, decimals, measurements, and scales.

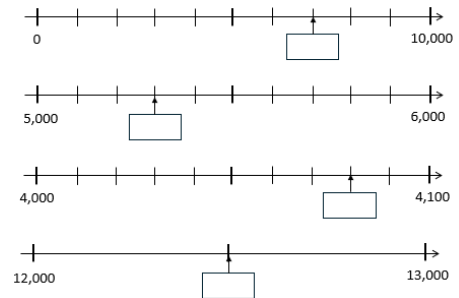
### What kind of errors and other warning signals can be expected with this task?

Some children count tick marks without understanding the scale, leading to errors (especially if the intervals are not 1). A common error is ignoring the role of zero in numbers (omitting or misplacing zeros). For example, in task 4a, they may write 700 instead of 7,000, or in task 4b, 5,030 instead of 5,300. Sometimes, the mistakes children make in this task indicate deeper misunderstandings about number sense or place value. For example, numbers may be placed in the wrong order (e.g., 70,000 before 10,000), indicating confusion about numerical order.

### What kind of support could be given to children who show deficits with this task?

If a child cannot make a reasonable judgement about where to place a number, this may indicate weak numerical understanding. Teachers need to determine whether the difficulty lies with counting (e.g., skipping numbers), misunderstanding place value (e.g., confusing 100 with 10), scaling (e.g., not understanding equal spacing), or dropping or misplacing zeros.

Children benefit from practicing tasks that involve placing numbers on a number line. The focus should be placed on partially filled number lines, which children must complete with the missing numbers. Color-coded benchmarks (e.g., 0, 10, 100) can reinforce understanding. It is also advisable to practice placing numbers without exact tick marks and to encourage children to estimate based on known benchmarks. Teachers can also ask children to explain their placements. Where possible, teachers can use interactive number lines that allow children to drag and drop numbers. Real-life contexts (e.g., timelines, rulers, temperature scales) can also be helpful for better understanding.



## Task 5: Halving numbers up to 10,000

### Key skill tested with this task

Fluency and conceptual understanding of halving large numbers by using efficient mental strategies, including place-value reasoning and regrouping.

### Why is this skill a key skill?

Halving larger numbers is fundamental for more complex division operations, as it provides a stepping stone to dividing by other numbers (e.g., even numbers) through repeated halving and unitising across place values. Students who have already developed the skill of halving numbers show stronger number sense and place value understanding, recognizing how digits behave across different place value positions when divided. Moreover, halving is essential for developing mental calculation strategies that extend beyond division. For example, halving and doubling techniques may be employed to simplify multiplication problems, find percentages (such as 50% and 25%), and work with fractions, also supporting proportional reasoning. Furthermore, halving has significant real-world applications that students encounter regularly. From sharing amounts equally between two people to calculating discounts, measuring cooking ingredients, or determining time intervals, the ability to quickly and accurately halve quantities is practically invaluable. When students achieve halving fluency, cognitive resources can be focused on higher-level reasoning and problem solving.

a) Half of 1,000: \_\_\_\_\_

b) Half of 500: \_\_\_\_\_

c) Half of 700: \_\_\_\_\_

d) Half of 3,000: \_\_\_\_\_

### What kind of errors and other warning signals can be expected with this task?

Computational errors are common. Students may incorrectly halve individual digits without considering place value, for example, interpreting "half of 3,000" as "half of 3, half of 0, half of 0, half of 0" instead of recognising 3,000 as 3 thousands. Some students make calculation mistakes with basic halving facts, such as stating that half of 1,000 is 50 instead of 500, indicating gaps in their understanding of how place value affects division. Place value misconceptions represent more serious conceptual difficulties. For instance, when halving 7,000 they might give answers like 350 instead of 3,500, showing they understand the arithmetic (half of 7 is 3.5) but cannot properly apply place value concepts to position the digits correctly. Warning signs include over-reliance on counting strategies or written algorithms for problems that should be solved mentally, such as using long division to find half of 1000. Inconsistent performance – correctly halving some numbers while struggling with similar problems – suggests weak conceptual understanding. Additionally, students who avoid these problems or express anxiety may lack confidence in their place value knowledge or basic halving facts. Visible hesitation when working with zeros in large numbers often reveals uncertainty about how place value affects division.

### What kind of support could be given to children who show deficits with this task?

Concrete and visual supports provide essential scaffolding for students who struggle with abstract concepts. Use place value materials such as base-ten blocks or place value charts to demonstrate how halving affects different place values. For example, show that halving 1,000 means taking 10 hundreds and dividing them into two equal groups of 5 hundreds each, resulting in 500. Alternatively, use number lines, arrays, or base-ten blocks to illustrate the relationship between halving and division by 2. Basic halving facts for single digits and multiples of 10 are important. Teach students to recognize patterns, which may help them apply known facts to larger numbers efficiently; for example, if they know half of 10 is 5, then half of 100 is 50, half of 1000 is 500, and so on. Integrate estimation benchmarks and comparative questions, using a "predict-then-check" estimation to evaluate reasonableness before computing, for example, "Is half of 700 closer to 300 or 400? Why?" Encourage multiple methods - unitizing on a chart, regrouping with blocks, or mental halving of known benchmarks – followed by brief reflections comparing efficiency and clarity. Use authentic contexts that naturally suggest halving and allow independent checking, such as "A recipe calls for 1000 g of flour; how much is needed for half a batch?" Invite students to design visual displays of halving patterns across powers of ten and to pose their own halving contexts, explaining how the context helps verify the result, thereby connecting computation, interpretation, and self-monitoring. Link halving to quarters and halving with doubling.

## Task 6a: Mental calculation: Addition and subtraction

### Key skill tested with this task

Fluency in applying mental calculation strategies for addition and subtraction, using compensation methods, near number strategies, and place value understanding.

### Why is this skill a key skill?

Mastering mental calculation strategies is fundamental for advanced mathematical learning, as students develop the numerical flexibility essential for tackling complex multi-step problems, algebraic reasoning, and proportional thinking in later years. Mental calculation strategies significantly enhance problem-solving efficiency by reducing cognitive load, allowing students to focus on understanding problem contexts, selecting appropriate operations, and engaging more deeply with mathematical reasoning. Moreover, these strategies are linked to students' conceptual knowledge of numbers and their interrelations, number sense, and place value (e.g., when students recognise that  $248 + 52$  can be solved by thinking " $248 + 2 + 50$ " or even " $(250 - 2) + (50 + 2)$ ", or that  $637 + 99$  becomes easier as " $637 + 100 - 1$ "), forming the foundation for later learning involving decimals, fractions, and proportional reasoning. Finally, the ability to quickly and accurately perform mental calculations using compensation and estimation strategies proves invaluable in various everyday practical situations.

### What kind of errors and other warning signals can be expected with this task?

Computational errors occur when students struggle with compensation strategies. They may identify 99 as close to 100 but forget to adjust their final answer, calculating  $637 + 99$  as  $637 + 100 = 737$  without subtracting 1. Students may also make arithmetic errors with adjusted numbers, such as miscalculating  $248 + 50$  or  $723 - 20$ , showing gaps in foundational number facts that compound strategic challenges. Strategy selection errors indicate conceptual difficulties. Students may apply inappropriate strategies, using compensation when simpler approaches work better or failing to recognize when numbers are close enough to multiples of 10 or 100 for adjustment. Others use inefficient counting strategies or attempt to replicate written algorithms mentally, leading to errors and increased cognitive load. Place value misconceptions become apparent when students example, when solving  $453 - 99$ , students might subtract 100 and add 1 to get 354, instead of applying the correct struggle to understand how adjustments affect digit positions. Inconsistent performance, avoidance of mental strategies, or expressing anxiety about "doing maths in their head" may indicate weak number sense foundations.

### What kind of support could be given to children who show deficits with this task?

Explain the hierarchy of calculation competencies and the dependence on place value understanding. Use number lines to illustrate compensation methods, such as showing how  $637 + 99$  can be represented by jumping to 737 ( $637 + 100$ ) and then stepping back one. Place value charts help students understand how adjustments across digits when working with numbers like  $453 - 99$ . Base-ten blocks can model "friendly numbers," demonstrating why  $248 + 52$  becomes easier when rewritten as  $250 + 50$ . Teach students to recognise "near numbers" and when compensation strategies are most effective. Help them identify numbers close to multiples of 10 or 100 (such as 199, 201, 98, 102) and practise the two-step process: adjust to create friendly numbers, then compensate in the opposite direction. Use problems designed for these patterns, such as  $248 + 52$  (near doubles),  $637 + 99$  (near hundreds), and  $723 - 24$  (near tens). Encourage students to ask, "Are these numbers close to multiples of 10 or 100?" and "Would adjusting help?" Teach the compensation steps: identify near numbers, decide on the adjustment, calculate with friendly numbers, and compensate in the opposite direction. Connect to real-life scenarios, for example, "A school trip costs 199€ per child. If 637 children attend, approximately how much is needed?" Progress practice from simple to complex, starting with single adjustments ( $37 + 19 = 37 + 20 - 1$ ) before moving to larger numbers. Use number talks to allow students to share strategies and develop mathematical flexibility. Combine formal and informal assessment, using mental mathematics sessions to evaluate strategy selection. Have students explain their reasoning to identify misconceptions. Creative projects, such as designing mental mathematics games or creating strategy posters, encourage deeper engagement and mathematical communication.

- |    |                                |
|----|--------------------------------|
| a) | $248 + 52 = \underline{\quad}$ |
| b) | $637 + 99 = \underline{\quad}$ |
| c) | $723 - 24 = \underline{\quad}$ |
| d) | $453 - 99 = \underline{\quad}$ |

## Task 6b: Mental calculation: Dealing with zeroes

### Key skill tested with this task

Fluency in applying place value knowledge for mental calculations with multiples of hundreds and thousands, focusing on digit operations in place value positions and zeroes as placeholders during addition and subtraction.

a)  $3,600 + 900 = \underline{\hspace{2cm}}$

b)  $56,000 + 8,000 = \underline{\hspace{2cm}}$

c)  $3,200 - 700 = \underline{\hspace{2cm}}$

d)  $54,000 - 5,000 = \underline{\hspace{2cm}}$

### Why is this skill a key skill?

Mastering mental calculations with thousands and hundreds demonstrates the base-ten system and place value understanding, which is foundational for working with larger numbers, decimals, and algebraic reasoning. When students mentally manipulate numbers such as  $3,600 + 900$  or  $54,000 - 5,000$ , they grasp digit position value of each digit and how operations affect place values. These strategies improve computation and number sense. Thinking flexibly about numbers, for example, "36 hundreds plus 9 hundreds equals 45 hundreds" or "54 thousands minus 5 thousands," develops number sense. This fluency enables students to focus on reasoning rather than computation. These skills prepare students for formal written methods. Students who can visualize combinations of thousands and hundreds have a better understanding of column addition and subtraction. Mental strategies also support estimation and verification. Place value understanding with large numbers has practical applications, such as in money, population, distances, and manufacturing, in students' everyday lives.

### What kind of errors and other warning signals can be expected with this task?

Place value misconceptions present significant conceptual challenges. Students may treat digits independently, failing to understand their positions within numbers. For example, when calculating  $3,600 + 900$ , they might add  $3 + 9$  and  $6 + 0$  separately, resulting in answers like 12,600 instead of recognizing 36 hundreds plus 9 hundreds. Students often struggle to understand that 54,000 contains 54 thousands, viewing it as separate digits. Errors related to zero occur when students misunderstand zero's placeholder role in large numbers. They may lose or gain zeros inappropriately - calculating  $56,000 + 8,000$  as 5,680 or 640,000 instead of 64,000. Algorithm interference arises when students use written column methods without understanding, leading to mistakes with different-digit numbers or zeros. Warning signs include confusion about combining numbers, such as stating that  $3,200 - 700$  involves "subtracting 7 from 3" rather than recognizing it as 32 hundreds minus 7 hundreds. Students who provide inappropriate answers demonstrate gaps in number sense and place value understanding. Those who avoid these problems or rely on inefficient counting strategies often lack confidence in their place value foundations.

### What kind of support could be given to children who show deficits with this task?

Support strategies should develop conceptual understanding of place value and calculation fluency. Visual aids help students grasp place value with large numbers. Use base-ten materials to show how thousands and hundreds combine. For example,  $3,600 + 900$  can be shown using 36 hundred-squares plus 9 hundred-squares to make 4,500 visible. Number lines in hundreds or thousands help students visualize calculations and understand magnitude. Unitising language helps students recognize patterns. Teach students to identify units: "3,600 is 36 hundreds, 900 is 9 hundreds, so 36 hundreds plus 9 hundreds equals 45 hundreds, which is 4,500." For  $54,000 - 5,000$ , emphasize "54 thousands minus 5 thousands equals 49 thousands." This language connects numbers to quantities and supports calculations. Progressive skill building should start with simpler examples before increasing complexity. Begin with hundreds before thousands. Use practice sequences showing patterns (e.g., comparing  $36 + 9 = 45$  with  $360 + 90 = 450$ ,  $3,600 + 900 = 4,500$ ). Estimation helps identify unreasonable answers (e.g., "3,600 is close to 4,000, 900 is close to 1,000, so the answer should be close to 5,000"). Encourage checking using inverse operations. Real-world contexts make calculations meaningful. Present scenarios such as: "A factory produces 54,000 components weekly. This week they make 5,000 fewer. How many do they make?" Present incorrect solutions and ask students to find errors. Extension challenges can engage confident students while reinforcing concepts. Present problems such as "Calculate  $3,600 + 900 - 700$ " or explore patterns like "If  $54,000 - 5,000 = 49,000$ , what is  $54,000 - 4,000$ ?"

## Task 7a: Written addition

### Key skill tested with this task

Fluency in applying formal columnar addition with numbers up to three digits, including place value alignment, regrouping, and systematic column-by-column calculation procedures.

### Why is this skill a key skill?

Mastering formal written methods for addition is essential in primary mathematics education in some countries. These methods provide reliable strategies for calculations with larger numbers that cannot be solved mentally. When students perform column addition, they develop tools necessary for multi-step problem solving.

Formal written methods, when appropriately taught, can reinforce understanding of place value relationships in the base-ten system. Aligning digits, regrouping between place values, and recognising operations strengthen conceptual understanding of number structure. Place value fluency prepares students for decimals, fractions, and algebraic reasoning. These algorithms provide verification mechanisms and alternative solutions. Students who master both mental and written methods can select appropriate strategies and verify answers using different approaches. This flexibility enhances problem-solving confidence.

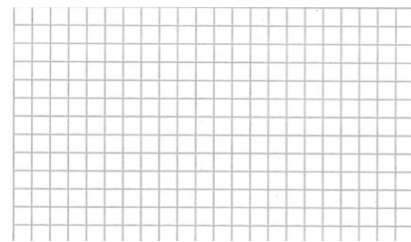
### What kind of errors and other warning signals can be expected with this task?

Place value and alignment errors are fundamental issues that affect calculation accuracy. Students may misalign digits when setting up calculations – for example, placing  $548 + 36$  with the 6 under the 4 instead of the 8. Difficulties with understanding column place values can cause confusion about digit positioning during regrouping. Regrouping misconceptions arise when students misunderstand exchanges between place values. Common errors include forgetting to carry when sums exceed nine or carrying incorrect amounts. Algorithmic errors occur when students apply procedures incorrectly, such as working from left to right instead of right to left, or confusing operations. Some students apply rules inappropriately, for example, always carrying 1. Warning signs include difficulty with numbers containing zero (e.g.,  $806 + 534$ ) and producing inappropriate answers, which indicate poor number sense. Students who avoid written methods may lack confidence in foundational skills.

### What kind of support could be given to children who show deficits with this task?

Concrete and visual representations provide essential scaffolding for students to develop an understanding of formal written methods. Use base-ten blocks to model exchange during regrouping, showing how 10 ones become 1 ten when calculating  $548 + 36$ . Place value charts help students visualise column alignment and understand digit positioning. Visual models such as expanded notation bridge the gap between mental strategies and algorithms by showing how  $548 + 36$  relates to  $(500 + 40 + 8) + (30 + 6)$ . Systematic instruction should emphasise a logical sequence while connecting actions to mathematical reasoning. Teach students routines: align digits by place value, start from ones column, calculate sums, and regroup as needed. Use consistent language and encourage students to verbalise their thinking. Guided practice with error analysis helps students recognise misconceptions. Present worked examples with errors and ask students to identify mistakes – for example, showing  $548 + 36 = 574$  and discussing why this is incorrect. Progressive skill building should move from simple to complex calculations. Begin with additions requiring no regrouping, but quickly move to single-exchange problems, and progress to multiple regrouping steps. Real-world problems make formal methods meaningful. Scenarios such as, "A school is organising a sponsored walk. Grade 3 raises €548 and Grade 4 raises €36. How much have they raised?" Help students grasp calculations. Teach students to estimate before calculating and to check using inverse operations. Extension activities can challenge confident students while reinforcing concepts through multi-step problems or by explaining methods to younger students. For students needing support with place value concepts, use structured resources such as NCETM's Column Addition and Subtraction materials for practice and demonstrations.

a)  $548 + 36$                       b)  $760 + 564$



## Task 7b: Written subtraction

a)  $711 - 67$

b)  $806 - 534$



### Key skill tested with this task

Fluency in applying formal columnar subtraction with numbers up to three digits, including place value alignment, regrouping, and systematic column-by-column calculation procedures.

### Why is this skill a key skill?

Mastering formal written subtraction methods is essential in primary mathematics education in some countries. These methods enable accurate computation with larger numbers in multi-step problems. When taught appropriately, they can reinforce understanding of place value in the base-ten system. Aligning digits, regrouping, and recognizing operations strengthen conceptual understanding. Fluency in place value prepares students for decimals, fractions, and algebraic reasoning. These algorithms provide verification and alternative solutions. Mastery of written subtraction complements mental methods by offering an alternative approach and checking mechanism, enhancing strategic choice and accuracy.

### What kind of errors and other warning signals can be expected with this task?

Place value and alignment errors affect calculation accuracy. Students may misalign digits when setting up calculations, such as placing  $981 - 37$  with the 3 under the 9 instead of the 8. Regrouping misconceptions may cause students to forget exchanges when the top digit is smaller, borrow from the wrong columns, fail to decrement after borrowing, or attempt to "carry" in subtraction as in addition. Students may subtract smaller digits from larger ones regardless of position, indicating confusion about subtraction direction and exchanging. Problems arise with zero-containing minuends, leading to skipped exchanges, lost zeros, or unchanged higher digits after borrowing. Algorithmic errors occur when students apply procedures incorrectly, such as working left to right instead of right to left, mixing addition and subtraction steps, or mishandling borrowed digits. Warning signs include difficulty with zero-containing numbers (e.g.  $806 - 534$ ), reliance on counting back instead of column work, inconsistent handling of similar problems, anxiety about layout, and lack of estimation to check results.

### What kind of support could be given to children who show deficits with this task?

Concrete representations scaffold students' understanding of formal written subtraction methods; use base-ten blocks or place-value counters to model exchange during regrouping, demonstrating how 1 ten becomes 10 ones when the top digit is smaller (for example, in  $806 - 534$ , exchange 1 ten to complete the ones subtraction). Place-value charts help visualize column alignment and digit positioning; visual models such as expanded notation bridge mental strategies by showing how  $806 - 534$  relates to  $(800 + 0 + 6) - (500 + 30 + 4)$  and where exchanges occur. Systematic instruction should emphasize logical steps while connecting actions to reasoning; teach routines: align digits by place value, start from the ones column, compare digits, regroup when necessary, subtract within the column, record the result, and adjust the next column for exchange. Guided practice with error analysis helps students recognize misconceptions; present worked examples with typical errors and ask students to identify corrections (for example, showing a solution to  $760 - 564$  that subtracts the smaller from the larger digit regardless of position and discussing why correct regrouping yields 196). Progressive skill building should move from simple to complex calculations. Begin with subtractions requiring no exchange but quickly progress to those needing borrowing from one or multiple columns, involving chained regrouping and zeros (for example,  $1,200 - 487$ ). Real-world contexts make formal methods meaningful. Present scenarios such as: "A cinema has 760 seats. 564 tickets are sold. How many seats remain empty?" Teach students to estimate before calculating (for example,  $806 - 534$  should be between 200 and 300). Encourage checking using inverse operations. Extension activities can challenge students while reinforcing concepts through multi-step problems requiring several calculations or exploring methods with larger numbers. For students needing support with place value concepts, use structured resources such as NCETM's Column Addition and Subtraction materials for practice and demonstrations.

## Task 8: Operational understanding of addition/subtraction

### Key skill tested with this task

Understand and solve arithmetic problems involving comparison by difference. In this case, involving ages.

David is 35 years old. He is 4 years older than Helen. How old is Helen?

My calculation:

My answer: Helen is \_\_\_\_\_ years old.

### Why is this skill a key skill?

This skill demonstrates specific mathematical abilities. It enables students to translate real-world situations into mathematical operations, linking abstract concepts to everyday life. It develops essential subtraction skills and logical reasoning, including interpreting comparative terms such as "more than," which supports both mathematical and language skills. Applying this skill to familiar contexts, such as calculating ages, makes learning more relevant. Ultimately, it sharpens mathematical thinking and provides transferable problem-solving tools, forming a core component of numerical literacy that supports future learning in mathematics and other areas.

### What kind of errors and other warning signals can be expected with this task?

Students often struggle with age-difference problems, revealing significant gaps in understanding. A common error is misinterpreting comparative language – such as assuming "more than" means to add rather than subtract – which leads to incorrect operations. Even when the correct operation is chosen, calculation errors in basic subtraction indicate broader issues with arithmetic fluency. Deeper conceptual issues are also evident. Some students use incorrect operations such as multiplication or division, showing a disconnect between the problem and their chosen strategy. Others struggle to explain their reasoning, indicating limited metacognitive awareness. The persistent use of concrete counting methods (for example, fingers or drawings) for problems that require mental calculation often points to underdeveloped number sense and delayed abstract thinking. These patterns highlight foundational gaps that require targeted instruction to develop both conceptual understanding and procedural fluency.

### What kind of support could be given to children who show deficits with this task?

To support students who struggle with additive comparison problems, it is essential to strengthen both their conceptual understanding of subtraction and their grasp of the relationship between subtraction and addition. Many difficulties arise from viewing subtraction only as "taking away," without understanding its meaning as a comparison between quantities or its connection to addition.

1. Use of manipulatives. This support focuses on understanding subtraction. Using objects such as blocks or counters helps students visualize what it means to be "4 years younger" as a difference between two quantities. Students can see physically that Helen's age must be smaller than David's, reinforcing subtraction as comparison rather than simple removal.
2. Visual representations (bar models). This support addresses both understanding subtraction and its relationship with addition. Bar models show David's age (35) as one bar and Helen's age as a shorter bar ( $35 - 4$ ). Seeing the difference visually – and noticing that Helen's bar can be obtained by subtracting, or alternatively, that adding 4 returns to David's bar – clearly connects the two operations.
3. Verbal scaffolding (guiding questions). This support focuses on understanding subtraction. Questions such as "Who is older?", "Should Helen's age be more or less?", and "Which operation expresses the difference?" help students identify that the problem describes a comparative relationship. This guidance helps them choose subtraction as the operation that expresses that difference.
4. Inverse check (verification through addition). This support focuses on the relationship between addition and subtraction. Encouraging students to check their answer by asking, "If Helen is 31, does  $31 + 4 = 35$ ?" reinforces the idea that addition and subtraction are inverse operations. This verification consolidates the understanding that both operations express the same relationship from different directions.

## Task 9: Basic number facts multiplication

### Key skill tested with this task

Fluency and automaticity in recalling basic multiplication facts (0-10)

### Why is this skill a key skill?

Mastering basic multiplication facts is essential for several reasons. First, it provides a solid foundation for advanced mathematics. When students can quickly recall multiplication facts, they perform multi-digit multiplication and division more efficiently and approach topics such as fractions and algebra with greater ease. Second, it increases problem-solving efficiency by reducing cognitive load. Instead of focusing mental energy on basic calculations, students can focus on selecting and applying appropriate problem-solving strategies. Finally, multiplication fluency has significant real-world applications, from calculating costs and measurements to managing time effectively. The ultimate goal is to achieve automaticity – instant recall of facts – as continued reliance on counting or repeated can hinder progress and pose challenges when tackling more advanced mathematical concepts.

a)  $6 \times 1 = \underline{\quad}$

b)  $10 \times 8 = \underline{\quad}$

c)  $8 \times 4 = \underline{\quad}$

d)  $7 \times 9 = \underline{\quad}$

e)  $9 \times 0 = \underline{\quad}$

f)  $7 \times 5 = \underline{\quad}$

### What kind of errors and other warning signals can be expected with this task?

Students learning multiplication may display certain errors and warning signs that indicate difficulties with mastering facts. A common error is producing incorrect products, such as misapplying known facts – for example, answering  $7 \times 5 = 30$  instead of 35 or confusing related facts like  $7 \times 9 = 63$  and  $8 \times 9 = 72$  (answering  $7 \times 9 = 72$  and  $8 \times 9 = 63$ ). Another frequent issue is slow response time, where hesitation or reliance on finger-counting occurs even for simple facts like  $3 \times 2$ . Some students may also skip items entirely or guess answers, producing random numbers instead of calculated results.

Important warning signs to watch for include an overreliance on additive strategies, such as repeatedly adding numbers instead of recalling a multiplication fact. Inconsistent accuracy – for example, knowing  $5 \times 4$  but not  $5 \times 6$ —indicates an unstable understanding. In other words, their understanding of multiplication is not yet fully consolidated; the student understands the concept only partially or uncertainly and still needs to reinforce their knowledge to gain confidence and accuracy in all multiplications. Finally, visible anxiety, stress, or avoidance behaviours are strong indicators that multiplication is a significant source of frustration or lack of confidence.

### What kind of support could be given to children who show deficits with this task?

To support students who struggle with basic multiplication facts, it is essential to structure support progressively, ensuring conceptual understanding first and then automation, based on relationships between known facts.

1. Check operational understanding. This support ensures the student understands what multiplication means before memorizing tables. For example: Do they know that  $7 \times 5$  means “7 groups of 5”? Can they represent it with drawings, blocks, or real-life situations? Avoid moving on if the student only adds by memory without understanding the operation.
2. Check automation of key facts. Some multiplications are fundamental and easier to automate, so first it is useful to check the multiplication tables of 2, 5, and 10. Ensure the student knows them quickly and without errors. If they are not automated, reinforce them before moving on to other multiplications.
3. Work on deriving other facts from key facts. Once the main tables are mastered show how to obtain other multiplications from known facts. For example, if they know  $5 \times 5 = 25$ , then  $6 \times 5 = 25 + 5$ . If they know  $8 \times 4 = 32$ , then  $4 \times 8 = 32$  (commutative property). This reduces the memorization load and fosters understanding of relationships between operations, allowing flexible strategies.
4. Automation based on understanding. Finally, reinforce all tables so they are fast and reliable, but always based on understanding the relationships between facts:
  - Easy combinations derived from known ones.
  - Use of patterns and properties (commutative property, multiplication by 0 and 1).
  - This ensures that automation is solid and flexible, not just mechanical memory.

## Task 10: Basic number facts division

### Key skill tested with this task

Fluency and conceptual understanding of basic division facts (1-10).

### Why is this skill a key skill?

Mastering basic division facts is essential for several reasons. It provides a strong foundation for advanced mathematics, supporting learning in areas such as fractions, long division, algebra, and problem-solving with ratios. It also strengthens division fluency by reinforcing the inverse relationship between the two operations, helping students develop a reciprocal understanding of how they are connected. Beyond the classroom, division is used in everyday situations, from sharing items equally to calculating rates. True fluency is demonstrated by the automatic recall of these facts, without relying on counting strategies or repeated multiplication checks.

a)  $80 : 10 = \underline{\quad}$

b)  $6 : 6 = \underline{\quad}$

c)  $28 : 4 = \underline{\quad}$

d)  $72 : 9 = \underline{\quad}$

e)  $30 : 5 = \underline{\quad}$

f)  $7 : 1 = \underline{\quad}$

### What kind of errors and other warning signals can be expected with this task?

Students often struggle with fundamental division concepts in predictable ways.

- A frequent mistake is **operation confusion**, where they multiply instead of divide (e.g., solving  $30 \div 5$  as  $30 \times 5 = 150$ ).
- Other issues include **incomplete fact recall**, particularly with less common divisors (e.g.,  $72 \div 9$  mistaken as 9), and misapplying **identity/zero rules** ( $7 \div 1$  confused with  $7 \times 0$ ).

These patterns reveal both computational gaps and underlying conceptual misunderstandings of inverse operations that require targeted intervention.

### What kind of support could be given to children who show deficits with this task?

To support students who have difficulties with basic division, assistance can be organized progressively: first ensuring conceptual understanding of multiplication, then the automatization based on understanding.

1. Check operational understanding of multiplication. Before working on division, ensure the students understand multiplication: Do they know that  $10 \times 8 = 80$  means “10 groups of 8” or “8 groups of 10”? Can they represent this with blocks, drawings, or real-life situations (for example, 8 bags with 10 sweets each)? This provides the necessary foundation to perform divisions correctly.

2. Check automatization of key multiplication tables. Ensure the student has mastered the most important tables: 2, 5, and 10, followed by the others. Basic division depends on knowledge of multiplication: for example, to solve  $72 \div 9$ , they must recognize that  $9 \times 8 = 72$ . If the tables are not automatized, reinforce them before moving on to division.

3. Introduce conceptual understanding of division. Explain that division is the inverse operation of multiplication: “If  $10 \times 8 = 80$ , then  $80 \div 10 = 8$  and  $80 \div 8 = 10$ .” Use visual representations: group diagrams, arrays, or drawings showing how to divide objects into equal parts. Have the student verbalize what division means: “Distribute 80 sweets equally into 10 bags. How many sweets are in each bag?”

4. Work on deriving division facts from known multiplication. Connect divisions to already mastered multiplications:

- $24 \div 4 \rightarrow “4 \times ? = 24” \rightarrow 4 \times 6 = 24 \rightarrow$  answer 6.
- $36 \div 6 \rightarrow “6 \times ? = 36” \rightarrow 6 \times 6 = 36 \rightarrow$  answer 6.
- This reduces the memorization load and helps students understand that division can always be derived from multiplication.

5. Automatization based on understanding. Reinforce all basic divisions until they are fast and reliable, always based on understanding multiplication and the inverse relationship. Practise patterns and properties: division by 1, division by the same number (result 1), relation to multiplication by 0. This ensures that automatization is solid, flexible, and functional.

## Task 11: Mental calculation: dealing with zeroes

### Key skill tested with this task

This task assesses students' ability to perform mental multiplication and division with numbers that are multiples of ten, one hundred, one thousand, and so on.

### Why is this skill a key skill?

A sound understanding of the decimal place value system forms the basis for flexible calculation with multi-digit numbers (and later, decimal numbers), and for relating these numbers to each other and to the world around us (for example, to estimate, make rough calculations, or correctly assess quantitative proportions in real-life situations). Understanding place value is multi-faceted. If a number ends with a zero, calculations can be performed using the whole number of tens, drawing on knowledge of smaller numbers. If the number ends with two zeros, calculations are performed on hundreds; with three zeros, on thousands, and so on. For example,  $7 \times 5,000$  is 7 times 5 thousand, which is 35 thousand, written as 35,000. We can also divide it into  $7 \times 5 \times 1000$ . This enables mental calculation with large numbers. This skill can be used to estimate the order of magnitude of a calculation. For example, instead of calculating  $7 \times 4,957$ , you can calculate  $7 \times 5,000$  by doing  $7 \times 5$  thousand, and obtain 35,000 as the order of magnitude.

### What kind of errors and other warning signals can be expected with this task?

Reasoning with number units (tens, hundreds, thousands, etc.) can be automated by manipulating zeros. However, this manipulation can produce results "with zeros" that are incorrect, especially when applied to decimals. For example, giving 2.30 or 20.3 as the result of the calculation  $2.3 \times 10$ . It is therefore important not to automate "zero" rules without understanding the reasoning behind them. A lack of response may indicate that the student is attempting to perform the operations mentally, as they would on paper (DiToM-Task 7). This technique is not the focus here, it is more difficult to apply and prone to errors.

### What kind of support could be given to children who show deficits with this task?

As stated, place value understanding is multi-faceted. If a child has difficulties with this task, a more comprehensive assessment of their current level of learning of the principles of the decimal system is recommended. There are two aspects to consider. The first is the decimal aspect, which involves recognizing that a unit of numbering (single unit, ten, hundred, etc.) of a certain rank is ten times that of the lower rank: 10 single units = 1 ten; 10 tens = 1 hundred; 10 hundreds = 1 thousand, and so on. The second aspect is the positional aspect: the order of the digits provides information about the units of measurement in question. Thus, while "2 tens and 3 units" does refer to a number, only "23" represents this number, while "32" refers to "3 tens and 2 units."

Depending on the results of such a detailed clarification of the learning level, it may be necessary to work with the child again on a fundamental understanding of the bundling principle. Important here are tasks in which the child forms decimal bundles with suitable materials. On this basis, a focus should be set on specific tasks where necessary, such as in DiToM-Task 3, 6 and 7. Short exercises can then be given daily, following a didactic progression based on the knowledge involved (decimal aspect, positional aspect). You can start with two-digit numbers with single units (u) and tens (t). For example: 3t 2u; 3t; 5u; 4u 2t; 4t 13u; 15u 3t; 35u 2t. For 15u 3t, the expected procedure is not a calculation ( $15 \times 1 + 3 \times 10$ ), but the following: since 15u is 10u 5u, then 15u is 1t 5u (because  $10u = 1t$ ), so we get 1t 5u 3t, which is 5u 4t, written as "45". For three-digit numbers, the equality  $10t = 1h$  will also apply. We can consider the following progression: 5c 3t 2u; 6c 4u 2t; 1c 2t; 3c 15u; 2c 4t 23u; 3c 15u 3t; 2c 14t 1u; 4u 21t 2c; 2c 14t 13u; 21t 15u 3c; 5c 9t 12u, and so on. The work continues with four-digit numbers and then others later. As always, it is important to help children progress to solving such tasks without materials, drawing on their previous experiences with material actions. For effective internalization, it is important to repeatedly ask children to verbalize their actions in an accompanying and increasingly anticipatory manner. Didactic tips for developing an understanding of the decimal place value system can be found, for example, at <https://mathe-sicher-koennen.dzlm.de/> or [https://padlet.com/frederick\\_tempier/numerationdecimale-la0yr0317fhb](https://padlet.com/frederick_tempier/numerationdecimale-la0yr0317fhb).

a)  $7 \times 5,000 =$  \_\_\_\_\_

b)  $50 \times 20 =$  \_\_\_\_\_

c)  $60,000 : 100 =$  \_\_\_\_\_

d)  $3,000 : 5 =$  \_\_\_\_\_

## Task 12: Operational understanding: Representations

### Key skill tested with this task

This task assesses students' ability to recognize the multiplicative structure of a representation.

### Why is this skill a key skill?

The task involves recognizing a multiplicative situation (eight sets of six dots each) and producing an arithmetic expression ( $8 \times 6$  or  $6 \times 8$ ) without calculating the result. More specifically, it requires translation beyond the arithmetic notation of repeated addition ( $6 + 6 + 6 + 6 + 6 + 6 + 6 + 6$ ) to produce an arithmetic notation corresponding to multiplication ( $8 \times 6$ ). This task is fundamental because it requires recognizing a multiplicative situation in order to identify the type of problem before performing the calculation. It enables students to perceive repeated addition as multiplication, which is necessary to:

- Build multiplication tables;
- Understand the meaning of the word "times" (in "32 times 100", which can be written as  $100 \times 32$  or  $32 \times 100$ );
- Understand the rules based on decimal notation when multiplying a number in the tens or hundreds by a given integer (the word "times" means that we add 32 times a hundred), rather than relying on rules about adding zeros to multiply by 10;
- Understand distributivity (13 times 5 is 10 times 5 plus 3 times 5, which can be worked out using repeated addition).

This task also allows us to assess the student's flexibility in switching from one representation to another, which requires a change of register. More specifically, it involves switching from a representation in the symbolic register to a representation in the register of arithmetic expressions.

### What kind of errors and other warning signals can be expected with this task?

#### When the situation is not understood:

- Incorrect answer 8: the 8 sets of 6 dots each may be identified globally, but without perceiving that each dice itself contains a constellation of 6 dots.
- Incorrect answer 6: the 6 dots of the constellation of six in a dice could be perceived without taking the 8 dices into account.

#### When neither the additive nor the multiplicative situation is perceived:

- Correct answer 48: the student may use a counting procedure to count 48 dots.
- Incorrect answers 47 or 49 or others: the student may make a counting error given the numbers involved and the limited time allocated to complete the task.

#### When the additive situation is perceived but not the multiplicative situation:

- Unexpected correct answer  $6+6+6+6+6+6+6+6$ : obtained by considering that there are 8 dices and that each dice is a constellation of 6 dots.
- Unexpected correct answer  $8+8+8+8+8+8$ : obtained from a constellation of 6 dots and, for each dot, considering that there are 8, since there are 8 sets of six dots each.

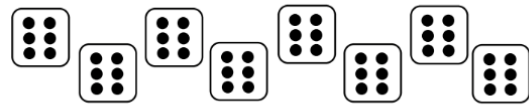
These answers, although correct, may show that the student remained stuck on repeated addition and did not translate it into multiplication.

- Incorrect answers  $6+6+6+6+6+6+6+6$  or  $6+6+6+6+6+6+6+6$ : the number of iterations may be incorrect.

#### When the multiplicative situation is perceived:

- Correct answers  $8 \times 6$  or  $6 \times 8$ : the student translates by multiplication that there are 8 sets of six dots each.
- Unlike other European countries, French school curricula do not distinguish between the meanings attributed to these two notations. French students can therefore use either  $8 \times 6$  or  $6 \times 8$  to express 8 sets of six dots each.

There is a calculation to figure out the total number of dots below.



Write down a multiplication task that fits the picture!  
You do not need to write the resulting total number of dots!

Calculation : \_\_\_\_\_

- Incorrect answers  $7 \times 6$ ,  $6 \times 6$  or  $9 \times 6$ : the number of occurrences corresponding to the number of dices with the six constellations may be incorrect.

**What kind of support could be given to children who show deficits with this task?**

Tips for the classroom:

- Present situations involving manipulation or verbalization like the one proposed, emphasizing what “multiplied by b” and “a times b” mean in everyday language.
- Vary the teaching variables to move from repeated addition to multiplication.
- Work on the commutativity of multiplication.
- Do not introduce the “ $\times$ ” sign too early.

This type of situation does not allow you to work on all the meanings that can be attributed to multiplication. For example, how many different menus can be created with three choices for a starter, two for the main course, and four for dessert? The result is obtained by a multiplication:  $3 \times 2 \times 4$ .

## Task 13: Operational understanding: Word problems

### Key skill tested with this task

This task assesses students' ability to recognize an additive / multiplicative model associated with a one-step arithmetic problem.

### Why is this skill a key skill?

The problems proposed are one-step problems, do not contain unnecessary data and are similar to those previously encountered by students at this level. The context (boxes of eggs for cooking) is intended to be familiar to all students, making it easier to understand the situation. The four arithmetic formulas proposed repeat the four operations with the same two numbers proposed in the problem statement, in the same order: first 24, then 6. The teacher's reading of the texts ensures that students with reading difficulties are not disadvantaged.

A baker buys 24 packets of eggs. In each packet there are 6 eggs. How many eggs does the baker buy?	<input type="text" value="24 : 6"/>
24 eggs are packed into packets. Each packet holds 6 eggs. How many packets get filled?	<input type="text" value="24 - 6"/>
There are 24 eggs in the fridge. The cook takes 6 eggs out of the fridge. How many eggs remain in the fridge?	<input type="text" value="24 x 6"/>
	<input type="text" value="24 + 6"/>

Recognizing the model underlying an arithmetic problem, especially in simple problems like the ones proposed, is a key skill for understanding operations and for solving more complex problems. It also ensures that students can identify these models in similar types of problems: a multiplicative situation involving finding the product (problem 1) or the number of parts (problem 2); an additive situation involving taking away/subtraction (problem 3). The aim here is to ensure that the student can model a problem by identifying the appropriate arithmetic entry, with an operation, without providing the result. The ability to calculate (which is a key skill assessed in other tasks) and to produce an answer sentence to the problem are not assessed.

### What kind of errors and other warning signals can be expected with this task?

An incorrect association between the text and the operation could be explained by:

- Poor understanding of the text or the situation: the student does not understand the meaning of a term (for example, confusing removing from the fridge with adding), or a word leads them to select the wrong operation (for instance, in French, the word "remain" in problem 3 may suggest a division problem).
- Failure to recognize the correct model or operation.

Some students may be able to solve problems 1 and 2 using repeated addition or subtraction; this indicates that they have not yet constructed multiplicative models (multiplication or division). Other students may solve all three problems using drawings, as the collections are small enough to be represented visually; in this case, the exercise shows that they are not yet able to produce arithmetic notation.

### What kind of support could be given to children who show deficits with this task?

As mentioned above, some students may solve the problems without using an operation; it therefore seems complementary to suggest that they do so without asking them to produce an operation. For example, the teacher could reintroduce multiplication or division as repeated addition or subtraction for a student who does not recognize multiplication or division, but who can solve problems 1 and 2 using repeated addition or subtraction.

If the student can solve the problem by drawing collections, this shows that the situation is understood. The three operations can then be reintroduced through verbalization, using the drawing and materials representing the collections in question. To help the student understand the situation better, the teacher can ask them to rephrase the problem in their own words or tell a story; the teacher can also mime the situation in front of the students (with eggs, boxes, tokens, etc.). There are also various ways to help students recognize the model underlying the problem: the teacher can provide materials to help represent the situation symbolically, ask students to relate the problem to one they have already encountered in class, or encourage them to estimate the order of magnitude of the result.

## 4) Notes on the Evaluation and Documentation of results

To help you evaluate the test results, various tools are available for download at: [ditom.org/en/tests-en](http://ditom.org/en/tests-en):

If you prefer to evaluate the tests manually, we provide the following aids:

- a) An **overview sheet for scoring**, which lists for each task the criteria for awarding one point, half a point, or no points (see page 43);
- b) A **class evaluation sheet** for recording and documenting the results of the entire class (see page 44);
- c) An **individual evaluation sheet** for recording and documenting the results of a single child, if you wish to keep an individual overview (see page 45);

A much less time-consuming option is to evaluate the results in Excel on your computer. For this purpose, you can download:

A **pre-programmed Excel file** with two worksheets that you can switch between via the tabs at the bottom left.

In the sheet titled “**qualitative**”, simply enter, in the appropriate column for each child, the numbers the child wrote in their test booklet as answers to each sub-task. If a child left an item blank, please enter 999.

When you have finished entering the data, switch to the “**quantitative**” sheet. The program will then automatically indicate whether each sub-task was answered correctly (1) or incorrectly (0) and will calculate the appropriate score for each overall task (1 / 0.5 / 0). At the end of each row, you’ll find the percentage of correctly solved tasks and the total score for the individual child. At the end of each column, you’ll find the percentage of children in the class who solved that particular task correctly.

### The “critical score thresholds” for DiToM 4+ and how to interpret them

As explained in Section 1, *DiToM* is not intended to label children. Please refer back to the discussion of *DiToM*'s goals and guiding principles in that section.

There you will also find a more detailed explanation of the “critical score thresholds,” which were determined based on pilot testing of *DiToM* (for version 4+, with 934 students across the project’s seven partner countries) using the statistical method of Latent Class Analysis. This method makes it possible to assign children, based on their total score in *DiToM* 4+, to one of the following three groups:

<b>Score Range</b>	<b>Group</b>
0 to 8.5	A - Signs of broad difficulties across several key areas
9 to 12.5	B - Indications of difficulties in some key areas
13 to 16	C- No indication of major difficulties in key areas

A final note referring back to Section 1: Keep in mind that a screening provides only a snapshot. The results should therefore be compared with your own classroom observations and experiences and, where indicated, used as a starting point for follow-up interviews with individual children — to deepen, refine, or expand your understanding, and, if necessary, to adjust your conclusions at least in part.

## Evaluation and Scoring *DiToM* Screening 4+ (max. 16 points)

1	Writing numbers	1 P. 0,5 P. 0 P.	all three numbers correct (5,089, 43,005, 300,500) two numbers correct all other solutions
2	Comparing numbers	1 P. 0,5 P. 0 P.	all three symbols correct (>, >, <) two correct all other solutions
3a	Adding 1/10/100 together with bundling	1 P. 0,5 P. 0 P.	all three numbers correct (9,900, 4,600, 4,000) two numbers correct all other solutions
3b	Taking away 1/10/100 together with unbundling	1 P. 0,5 P. 0 P.	all three numbers correct (6,999, 3,490, 3,900) two numbers correct all other solutions
4	Numbers on the number line	1 P. 0,5 P. 0 P.	all four numbers correct (7,000, 5,300, 4,080, 12,500) three numbers correct all other solutions
5	Halving numbers up to 10,000	1 P. 0,5 P. 0 P.	all four numbers correct (500, 250, 350, 1,500) three numbers correct all other solutions
6a	Mental calculation: Addition and subtraction	1 P. 0,5 P. 0 P.	all four numbers correct (300, 736, 699, 354) three numbers correct all other solutions
6b	Mental calculation: Dealing with zeroes	1 P. 0,5 P. 0 P.	all four numbers correct (4,500, 64,000, 2.500, 49,000) three numbers correct all other solutions
7a	Written addition	1 P. 0,5 P. 0 P.	both results correct (584, 1,324) one result correct all other solutions
7b	Written subtraction	1 P. 0,5 P. 0 P.	both results correct (644, 272) one result correct all other solutions
8	Operational understanding of addition/subtraction	1 P. 0,5 P. 0 P.	correct term and result ( $35 - 4 = 31$ ) either the task OR the result was noted correctly all other solutions
9	Basic number facts multiplication	1 P. 0,5 P. 0 P.	all six numbers correct (6, 80, 32, 63, 0, 35) five numbers correct all other solutions
10	Basic number facts division	1 P. 0,5 P. 0 P.	all six numbers correct (8, 1, 7, 8, 6, 7) five numbers correct all other solutions
11	Mental calculation: Dealing with zeroes	1 P. 0,5 P. 0 P.	all four numbers correct (35,000, 1,000, 600, 600) three numbers correct all other solutions
12	Operational understanding: Representations	1 P. 0 P.	correct multiplication ( $8*6$ OR $6*8$ ) all other solutions
13	Operational understanding: Word problems	1 P. 0,5 P. 0 P.	all three correct ( $a - 3$ , $b - 1$ , $c - 2$ ) two correct all other solutions



Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Evaluation form DiToM Screening 4+**

Item	Right answer	Check right/wrong	Points
1.a	5.089		
1.b	43.005		
1.c	300.500		
2.a	>		
2.b	>		
2.c	<		
3a.a	9.900		
3a.b	4.600		
3a.c	4.000		
3b.a	6.999		
3b.b	3.490		
3b.c	3.900		
4.a	7.000		
4.b	5.300		
4.c	4.080		
4.d	12.500		
5.a	500		
5.b	250		
5.c	350		
5.d	1.500		
6a.a	300		
6a.b	736		
6a.c	699		
6a.d	354		
6b.a	4.500		
6b.b	64.000		
6b.c	2.500		
6b.d	49.000		

Item	Right answer	Check right/wrong	Points
7a.a	584		
7a.b	1.324		
7b.a	644		
7b.b	272		
8 part 1	35-4		
8 part 2	31		
9.a	6		
9.b	80		
9.c	32		
9.d	63		
9.e	0		
9.f	35		
10.a	8		
10.b	1		
10.c	7		
10.d	8		
10.e	6		
10.f	7		
11.a	35.000		
11.b	1.000		
11.c	600		
11.d	600		
12	8*6 or 6*8		
13.a	a) - 3		
13.b	b) - 1		
13.c	c) - 2		

Total points achieved out of 16

Comment: \_\_\_\_\_

**Valuation:**

- |                      |  |
|----------------------|--|
| Items 1 to 3b and 13 | all 3 correct = 1 point; 2 correct = 0.5 points; 1,0 correct or missing = 0 points       |
| Items 4 to 6b and 11 | all 4 correct = 1 point; 3 correct = 0.5 points; 2,1,0 correct or missing = 0 points     |
| Items 7a to 8        | all 2 correct = 1 point; 1 correct = 0.5 point; 0 correct or missing = 0 points          |
| Item 9 and 10        | all 6 correct = 1 point; 5 correct = 0.5 points; 4,3,2,1,0 correct or missing = 0 points |
| Item 12              | correct = 1 point; wrong = 0 points  |

## 5)References

- Livingston, S. A. (2014). *Equating Test Scores (without IRT)*. 2<sup>nd</sup> edition. Educational Testing Service.
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