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DiToM
Diagnostic Tool in Mathematics

Screening 2+

Handbook for teachers

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Contents

Foreword

This manual is designed to support you in administering the DiToM 2+ screening and in using the test results effectively with your class. On the following pages, you will find:

1. a brief introduction to the aims and guiding principles of the Erasmus+ project *DiToM*;
2. detailed, step-by-step instructions for conducting *DiToM 2+* in the classroom;
3. concise explanations of each task in *DiToM 2+*, including notes on possible support strategies for children whose screening results indicate learning gaps in key mathematical competencies;
4. guidance on how to evaluate and document the results.

The administration guide (Section 2) and the evaluation tables (Section 4) can also be downloaded separately as individual PDF files from www.ditom.org/

We recommend printing the administration guide double-sided and spiral-binding it. In the booklet that you will get, you can keep the teacher-facing page for reading the instructions aloud, while the page facing the children often includes an example that helps you explain what the children are expected to do.

1 Aims and Guiding Principles of *DiToM*

Mathematics learning progresses in stages: new knowledge builds on secure prior understanding. When fundamental ideas and concepts are missing, students find it increasingly difficult to grasp and make sense of mathematical content that builds upon that foundation. Both national and international studies show that a significant proportion of students already fail to meet the minimum standards in mathematics at the primary level—and, for the reasons described above, almost inevitably continue to struggle in secondary school. Alarmingly, many young people finish their compulsory education without having achieved the basic level of mathematical literacy that, according to the OECD, is essential for "full participation in social life."

To counteract this, teachers must first be able to identify mathematical learning difficulties—ideally early and as precisely as possible. Only on this basis can targeted support measures be taken. This is precisely where the EU project *Diagnostic Tools in Mathematics (DiToM)* comes in. In a partnership between Germany, France, Greece, Croatia, Italy, Sweden, and Spain, five interconnected screening instruments were developed. These tools enable teachers, at the end or beginning of a school year, to gain a concise overview of which students are at risk of falling behind in mathematics if they do not receive targeted support measures.

The screenings follow a two-year cycle:

- **Screening 0** – Beginning of primary school
- **Screening 2+** – End of Grade 2 / Beginning of Grade 3
- **Screening 4+** – End of Grade 4 / Beginning of Grade 5
- **Screening 6+** – End of Grade 6 / Beginning of Grade 7
- **Screening 8+** – End of Grade 8 / Beginning of Grade 9

What are the *DiToM* Screenings and what do they achieve?

The five screenings are paper-and-pencil tests focused on key mathematical competencies that should be secure at the start of a grade level for new content to be learned with understanding. Each test can be administered to the whole class within a single lesson and, using the provided scoring tools (see Section 4), evaluated with relatively little time investment. The results give teachers an initial structured overview of which children are likely to need additional support in particular areas.

The word *"likely"* is crucial: a screening does **not** replace an individual, qualitative assessment of a child's learning status. At best, it provides initial clues as to what strategies or solution approaches a child may have used. More detailed understanding requires targeted observation and one-on-one discussions, using more finely differentiated tasks. The screening, however, can serve as a valuable starting point to determine which children would benefit most from such follow-up assessments.

What are "key mathematical competencies"?

As noted earlier, school mathematics is characterized by an *"internal learning hierarchy"* (Wittmann, 2015, p. 199). This is particularly true in the domains of arithmetic (numbers and operations) and algebra—the very areas that *DiToM* screenings intentionally focus on. In these domains, it is possible at every learning stage to identify *key competencies*—those without which further learning cannot take place meaningfully and sustainably.

For example: To work successfully with natural numbers, children must understand them in terms of the *part–whole concept*—a developmental process that should be completed during the first school year. The part–whole concept means, for instance, that the number seven is understood as a whole composed of parts—five and two, four and three, one and six, and so on. This understanding should then become automatic: a child should no longer need conscious effort to recognize five as the missing part of seven when two is given as the other part. In other words, children should automatically think of numbers in terms of their decompositions and relationships. This combination of *understanding* and *automation* is characteristic of many key competencies: only once certain skills are automatic can mental capacity be freed up to tackle higher-level mathematical challenges.

Whether the key competency of "thinking of numbers as compositions" (or "number decomposition") is well established can be seen, for instance, in a child's calculation strategies. A child who thinks of seven as five and two will solve 7 – 5 effortlessly, even in the first school year, without counting. Children who lack this competency, however, often continue to rely on laborious, error-prone counting strategies well into later primary and secondary grades. Counting-based addition and subtraction soon become unmanageable when two- or three-digit numbers are involved. Such children also struggle to use relationships between multiplication facts—for instance, recognizing that 9 × 6 is six less than the easily remembered 10 × 6. Deficits in one key competency (understanding numbers as compositions) thus hinder the acquisition of others (addition, subtraction, multiplication), which in turn are prerequisites for more advanced skills (division, proportional reasoning, etc.).

This chain continues beyond primary school: students who struggle with natural numbers will face even greater difficulties with fractions and decimals. Algebra, later on, builds on insights that should have been gained from working with the basic operations in primary school. Without those insights, algebra can appear to students as an indecipherable code.

For this reason, the *DiToM* screenings focus on key competencies—those that should be securely established at the start of Grades 1, 3, 5, 7, and 9, so that further mathematical learning can proceed successfully.

After administering the *DiToM* screening – what's next?

Using the evaluation tools described in Section 4, teachers create a table (Excel or paper) that can be read in two directions:

- **Across rows:** Each child's results show which tasks were solved correctly, partially correctly, incorrectly, or left blank—resulting in an overall score for that child.
- **Down columns:** For each task, the table shows how many children solved it correctly, partially correctly, incorrectly, or not at all.

With a view on individual students:

DiToM is not about labeling children. The screenings are **not** designed to identify students with "dyscalculia." Clinical diagnoses of that kind do not address the core question that *DiToM* seeks to answer: *How can teachers best support children struggling with key arithmetic competencies?* Targeted support requires an accurate understanding of each child's current learning level. *DiToM* helps identify those for whom such detailed assessment is urgently needed—nothing more, but also nothing less. Section 3 provides brief notes on what kinds of follow-up support may be helpful for each specific task.

The “critical threshold scores” given in Section 4 were determined based on trials of the *DiToM* screenings with 8,820 children across the seven partner countries. Using *latent class analysis* (see Livingston, 2014), children were grouped as follows:

- **Group A:** Children showing widespread difficulties across several key competencies.
- **Group B:** Children showing indications of difficulties in specific areas.
- **Group C:** Children showing no major indications of difficulty.

It is important to remember that any screening captures only a *snapshot*. Some children may simply have had a bad day or been distracted, others might have—despite precautions—copied answers. Screening results should therefore be interpreted cautiously. They should always be compared with observations from daily classroom, and used as a prompt for further targeted observation and follow-up tasks in the coming days and weeks.

If it becomes clear that a child falls into Group A, there is reason to expect that their mathematical difficulties will worsen over the school year unless timely and effective interventions are implemented. Chapter 3 can only suggest general directions for such interventions, based on the key competencies assessed by each task. For more extensive guidance, teachers must refer to the relevant educational literature.

Children in **Group B** are also likely to need targeted support in at least some areas to progress successfully in their learning. It is worth remembering that all screening tasks assess *key* competencies. The screening is intentionally designed *not* to distinguish among high achievers—ideally, most children should find the tasks quite easy. Therefore, any errors made by **Group C** children on individual tasks should also be taken seriously, as they may reveal gaps in key foundational skills.

With a view on the class as a whole:

The latter applies particularly when the results show that multiple children struggled with the same task. This may indicate that they have received insufficient or unfocused practice with that competency, either in their prior schooling or before entering school. In such cases, it is all the more important that these learning opportunities now be provided, even if the curriculum has already moved on to new content. Again, it is important to take into account the hierarchical structure of mathematics learning: each level depends on secure understanding of the foundational competencies before moving forward.

2 Instructions for Administering Screening 2+

Screening 2+ is designed for use with the entire class at the end of Grade 2 or immediately at the beginning of Grade 3.

It comprises the following tasks:

1. Cardinal counting
2. Tens–ones representations
3. Forwards and backwards in the number sequence
4. Writing two-digit numbers
5. Halving two-digit numbers
6. Numbers on number lines
7. Splitting numbers up to 10
8. Addition
9. Subtraction
10. Word problem 1 (addition)
11. Word problem 2 (subtraction)
12. Core multiplication facts
13. Interpreting a representation as multiplication
14. Word problem 3 (quotitive problems)
15. Word problem 4 (sharing)

The following section provides detailed, task-by-task instructions on what to tell the children before and during the administration of the test.

These instructions are also available as a **separate PDF file for download**, expanded with sample and blank pages for printing. If you print this file double-sided and bind it with a spiral, you will have a booklet from which you can read the instructions aloud during the test and refer back to key points to keep in mind during administration. The additional pages included in the print version allow you, by turning the left side of each double-page, to hold up the booklet and read the instructions from the page in front of you, while the children can see the corresponding example task on the back of the booklet.

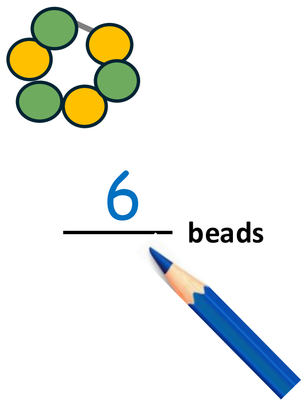
Before and During the Distribution of Test Booklets

- Explain to the children that at the end of Grade 2 / beginning of Grade 3 you would like to find out what they already know and can do.
- Tell them that each child will now receive a booklet with tasks to complete one after another.
- Emphasize that it is important for each child to work independently and that copying from a neighbor is not helpful. Another child's answer might be wrong — and most importantly, you want to know what each child can already do well on their own and where they might still need help.
- If necessary and possible, place school bags (or similar items) between children during the test to make copying more difficult.
- Ask the children to write with a pencil. Since erasing takes time, they should simply cross out any mistakes and write the correct answer next to them. You may wish to demonstrate this briefly on the board.
- Tell the children that the tasks are to be done one after another, and that you will always explain what to do before they start. Sometimes there will also be an example. Remind them not to continue on their own, even if they finish a task earlier than others. They should only turn the page when you tell them to do so.
- Explain that it's important for everyone to pay close attention and listen carefully to your instructions.
- Make sure all desks are empty and that each child has only one sharpened pencil in front of them.
- Some tasks have a **time limit**. To avoid stress, do not announce this in advance. Instead, tell them that you expect that they will solve some of the tasks rather quickly because they probably already know them by heart. Announce that when they have worked for some time on a task, you may say STOP, and then everyone should indeed stop writing. Emphasize that it is not a problem if someone has not finished at that point of time. For all the screening, the goal is a calm, stress-free environment.
- For **tasks without a time limit**, use your own judgement about when to say STOP. This may be advisable for some tasks, once most children have finished. Some children may take considerably longer than the big majority, and even with more time, might not complete the task. However, if others must wait too long, restlessness may arise. Therefore, it might be better to say STOP and assure those who have not finished that it does not matter, and praise the children for their efforts.

- **Now hand out the booklets.** Emphasize that they must remain closed on the desks until you tell the children to turn to the first task. Ask them first to write their name on the cover page.

1 Counting

Example



"Look at this bracelet. There are six beads in this bracelet.
Therefore, we write the number ***6*** down here."

→ *point to the line with the pen*

"There are six beads, so we write down 6, because there are 6 beads."

Screening task

No time limit.

_____ beads

"Now please turn your page over to the first task."

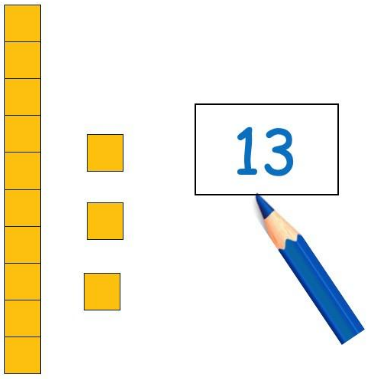
"Here you see another bracelet.
Count the beads in this bracelet ***quietly***!"

"Write the number of beads on the line below. Count silently and then write the number on the line."

"Once you are finished, please put your pencil on your desk."

2 Tens-ones-representations

Example



"Look at this picture. It shows the number ***thirteen***. ***Ten*** here and ***three*** here.
There are always 10 in a rod, so that is 10 and three single ones."

→ *point to the rod first and then to the cubes*

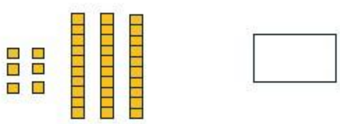
"So we have 10 and 3. Together this is 13. Therefore, we write the number ***13*** into the box."

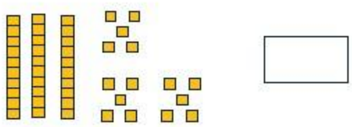
→ *point to the box and the pen*

Screening task

Time limit: 30 sec

a) 

b) 

c) 

"Please turn your page over to the next task."

"Here you see three other number pictures. Write each number in the box next to the picture. Start now."

→ *count to 30 in your head*

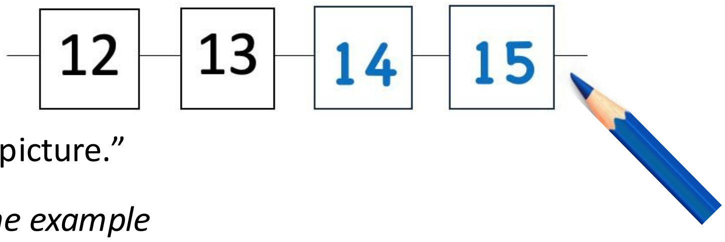
"We move on to the next task. It doesn't matter if you have not yet finished."

"Please look at this picture."

→ *point to the example for Item 3*

3 Forwards and backwards in the number sequence

Example



"Look at this picture."

→ *point to the example*

"There are four numbers in the row. It starts with twelve, thirteen and the number after that is ***fourteen***, this is why 14 is written in the next box. And after fourteen comes ***15***, so 15 is written in the following box."

→ *first point at 14 then at 15*

"The four numbers in this row are 12, 13, ***14*** and ***15***."

Screening task

No time limit.

"Now please turn your page over to the next task."

"Here, we always have five numbers in a row. Write the missing numbers in the empty boxes."

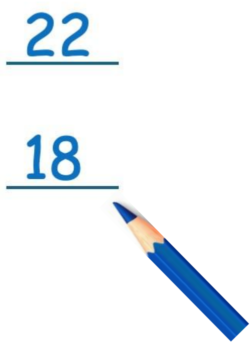
"Pay attention: Sometimes you will need to find the number that comes <u>before</u> another number!"

"Once you have finished, put your pencil down on the desk."

a)	37	38			
b)		87	88		
c)				61	62

4 Writing two -digit numbers

Example



"When we want to write 'twenty-two' we write it like this."

→ *point to the number 22*

"And when we want to write the number 'eighteen', we write it like this."

→ *point to the number 18*

Screening task

No time limit.

a) ____________

b) ____________

c) ____________

d) ____________

e) ____________

"Now I want you to write down more numbers."

"Please turn your page over to the new task. You see five lines a) to e), one below the other."

"I am telling you 5 numbers. One after the other. Listen carefully and write down the number:

a) thirty-four (34)
b) fifteen (15)
c) forty-three (43)
d) fifty (50)
e) sixty-seven (67)"

"Now, let us have a look at the next task."

5 Halving two-digit numbers

Example

Half of 10: 5

"Half of ten is five."

→ *point to the example task*

"This is why we write 5."

Screening task

Time limit: 30 sec

a) Half of 12: ______

b) Half of 16: ______

c) Half of 60: ______

d) Half of 80: ______

e) Half of 50: ______

"Please turn your page. You see five numbers. Write down what is ***half*** of these numbers."

"Start now!"

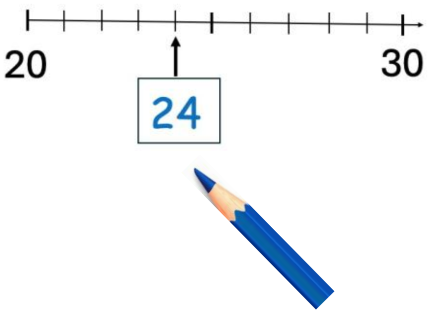
→ *count to 30 in your head*

"Let us move on to the next task.
It doesn't matter, if you could not finish."

→ *point to the example for Item 6.*

6 Numbers on number lines

Example



"Here you see the number line from 20 to 30."

→ *point it out by moving your finger along the line from 20 to 30*

"We are looking for the number that belongs in the box."

→ *point to the box*

"Check for yourself – it is the number 24. This is why 24 is written in the box."

Screening task

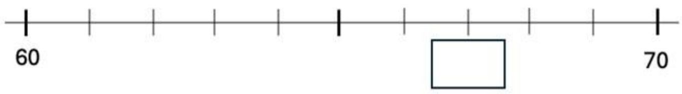
No time limit.

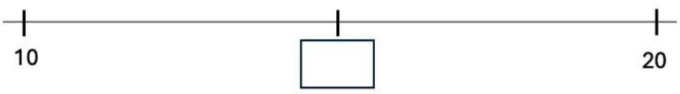
"Please turn the page over to find your new task. Here you see three different number lines."

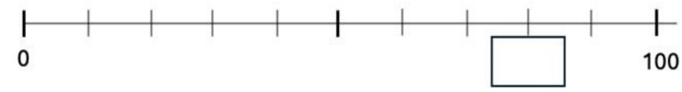
"Pay attention to numbers already written on each number line!"

„For each line, look carefully and write the correct number in the box."

"Once you are finished, put your pencil down please."

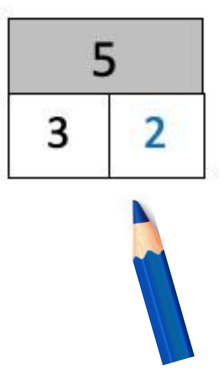
a) 

b) 

c) 

7 Splitting numbers up to 10

Example



"Here you see the number 5 in the top box."

→ *point your finger to the number 5*

"As you know, we can split the number 5 in two numbers. If one of the numbers is 3 …"

→ *point to the number 3*

"… then the missing number is 2, as 3 plus 2 is 5."

→ *point to the numbers as you speak*

"So, the number 5 can be split into the numbers 3 and 2. Together 2 and 3 make 5."

Screening task

Time limit: 30 sec

a)	b)	c)	d)	e)	f)
6	7	8	8	9	9
1	3	2	5	2	4

"Now please turn your page over. Here you find more numbers to split."

"Look carefully on the numbers at the top and then write the missing number in the empty box. The two numbers in the bottom add up to the number on the top."

"Please start now!"

→ *count to 30 in your head*

"Please stop. It doesn't matter, if you could not finish all numbers."

8 Addition

no example

Screening task

No time limit.

a) 32 + 7 =

b) 6 + 74 =

c) 60 + 30 =

d) 27 + 40 =

e) 25 + 8 =

"For the next task, we do not need an example, because you all know what to do. We are now doing ***addition***."

"Please turn over the page in your booklet and you will see some addition tasks."

"Do them now!"

"Once you have finished, please put you pencil down."

"So far you have done really good, and we have already done more than half of the tasks. Stand up and shake your arms, hands and legs for a moment. " → *you do the same*
"That feels good, right? Now please sit down again."

9 Subtraction

no example

Screening task

No time limit.

a) 48 – 6 =

b) 37 – 7 =

c) 20 – 9 =

d) 56 – 30 =

e) 25 – 8 =

“Now please turn your page and you will see some ***subtraction*** tasks.”

“We are doing subtraction now. Keep that in mind.”

“Now do the subtraction tasks!”

“Once you have finished, please put your pencil down.”

10 Word problem 1 (addition)

no example

Screening task

No time limit.

"Now please turn over the page and look at the next task I will read it for you."

→ *read the word problem* ***twice*** *to the class, stress the words in bold*

On the way to school:

There are **12** children on the school bus.

At the next stop, **6 more** children get on.

How many children are now on the bus?

My Calculation: ________________________

Answer: Now there are _________ children on the bus .

"Now do the problem. It is important that you also write your calculation with the result on the line. Then fill in the result in the answer."

"Once you have finished, please put down your pencil."

11 Word problem 2 (subtraction)

no example

Screening task

No time limit.

“Please turn over the page and look at the next problem.”

“Now school is out, and the bus takes the children back home.”

→ *read the word problem* ***twice*** *to the class, stress the words in bold*

On the way home:

There are **28** children on the school bus.

At the first stop, **3** children get off.

How many children are still on the bus?

My Calculation: ______________________

Answer: Now there are ________ children on the bus.

“Now do the problem. Again, it is important that you also write your calculation with the result on the line. Then fill in the result in the answer.”

“Once you have finished, please put down your pencil.”

12 Core multiplication facts

no example

Screening task

Time limit: 30 sec

a) $7 \times 2 =$

b) $4 \times 5 =$

c) $8 \times 10 =$

d) $9 \times 2 =$

e) $10 \times 7 =$

f) $5 \times 6 =$

"For the next task, we do not need an example, because you all know what to do."

"We are now doing multiplication."

"Please turn over the page in your booklet and you will see some multiplication tasks."

"Do them now!"

→ *count to 30 in your head*

"Please stop now. It does not matter, if you have not finished all the tasks. We will move on now."

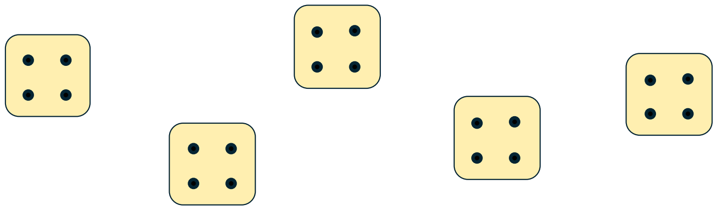
13 Interpreting a representation as multiplication

no example

Screening task

No time limit.

“Please turn over the page and have look at the next problem.”



____

“Look carefully at the picture. It shows a **multiplication** task.”

“Write this **multiplication** task on the line under the picture.”

“If you know it, you can also write down the result. But this is voluntary. It is more important that you write the matching task.”

“Once you have finished, please put down your pencil.”

14 Word problem 3 (quotitive problem)

no example as it would give away the solution strategy

Screening task

No time limit.

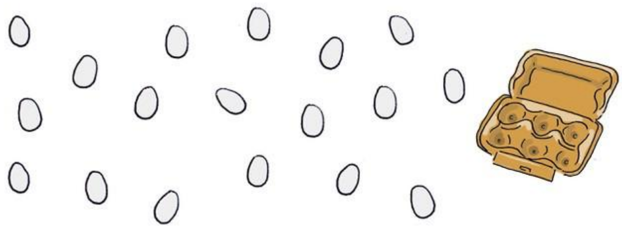
"We have two more problems left and so far, you have done great.
Let us look at the second last problem. Please turn over the page."

→ *read the word problem to the class **twice**, stress the words in bold*

This morning the farmer has picked up **18 eggs**.

6 eggs fit in an egg carton.

How many egg cartons can he fill?



Answer: The Farmer can fill ______ egg cartons.

"Now solve this problem. You can draw the solution using the picture or you can write down your calculation next to the eggs.
It is important that you write down your solution number in the answer."

"Once you have finished, please put your pencil down."

15 Word problem 4 (sharing)

no example as it would give away the solution strategy

Screening task

No time limit.

“Now please turn over to the last page. We have only one problem left.”

→ *read the word problem to the class **twice**, stress the words in bold*

Grandma has bought **15 chocolate eggs** to give them to her **3 grandchildren**.

Everyone get the same number.

How many chocolate eggs does each child get?

Answer: Each child gets _____ eggs.

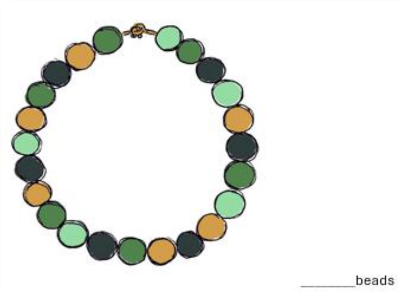
“Please solve this last problem. Again, you can draw the solution using the picture or you can write down your calculation next to the eggs. It is important that you write down your solution number in the answer.”

“Once you are finished, please put you pencil down, close your booklet and I will come around and collect it.”

→ *After collecting all the booklets: Thank the children for their hard work and cooperation and treat them with a run around the school yard or a game or whatever you find appropriate as a reward!*

3 Explanations and suggestions for support regarding the single tasks of *DiToM* 2+

Task 1: Cardinal counting



Key skill tested with this task:
Counting ordered quantities larger than 20

Why is this skill a key skill?
Learning the number word series and adhering to counting principles (see below) when determining a number by counting are important steps on the way to a sound understanding of natural numbers. Being able to determine the number of elements in a set by counting is a prerequisite for being able to discover and investigate relationships between numbers. The relationships "one more/one less" between two neighboring numbers in the number line and "part-whole" within so-called number triples (e.g. 3, 5 and 8; 8 the whole, 3 and 5 its parts) are fundamental. Of course, being able to count numbers accurately is also an important skill in everyday life.

What kind of errors and other warning signals can be expected with this task?
To correctly count the 23 beads in Task 1, a child must master the number word series up to "twenty-three" and adhere to the one-one principle when counting the beads. That a child has failed in one-to-one assignment, such as having omitted or double-counted individual beads, is particularly likely if s/he misses the correct number by one (e.g., 22 or 24). The arrangement of the beads in the circle also requires a planned approach, meaning the child must choose a starting bead and ensure that this one and any subsequent beads are not counted twice. The knot was deliberately included in the illustration to facilitate this, but the child must independently use it for this purpose.

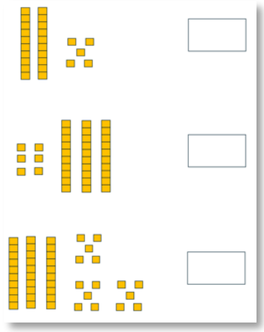
The task requires writing down the determined number using numerals. For example, the number "twenty-three" must be written as "23". Errors such as writing "32" (or "42" in combination with a counting error) could indicate a problem with writing two-digit numbers rather than counting (see Task 4).

What kind of support could be given to children who show deficits with this task?
The task was deliberately chosen as a starting task because children at the end of the 2nd school year will probably find it easy and actually rarely make mistakes. If a mistake is made, this does not necessarily mean that the child still has problems with the counting principles. Correct counting always requires concentration as well as an understanding of counting principles.

However, an error in task 1 should be a reason to check the child's counting skills more comprehensively outside of the screening. In doing so, you should go beyond the principles covered here (stable order and one-one principles): Is the child aware of the difference between cardinal (e.g. "eight beads" as the result of a count) and ordinal (e.g. "the eighth bead") use of number words? Does the child know that once a number has been determined by counting, it does not change (and therefore does not need to be recounted) if the order of the elements is changed? Do they realize that it doesn't matter whether they count from left to right (or clockwise or anticlockwise) as long as each element is counted exactly once? If there are still uncertainties in such questions at the end of the 2nd school year or later, it is urgently necessary to work with the child on basic counting skills.

If errors indicate problems with writing two-digit numbers with numerals (see above), the performance of the same child in Task 4 could provide further clues. In the commentary to Task 4 you will find information on possible support measures for children who have difficulties in this area.

Task 2: Tens-ones representations



Key skill tested with this task:
Perception of structured representations of two-digit numbers, bundling.

Why is this skill a *key* skill?
A sound understanding of the decimal place value system ist the basis for being able to calculate flexibly with multi-digit numbers (later also decimal numbers) and to relate these numbers to each other and to the world in which we are living (e.g., to estimate, make rough calculations, correctly assess quantitative proportions in real-life situations…).

Understanding of the decimal system is multi-faceted. To operate successfully with two-digit numbers, learners must distinguish *tens* and *ones* and understand that one ten is equivalent to ten ones (bundling principle). Connected with this, they must understand the *place value* of single numerals within a written two-digit number. Given the number 25, the face value 2 represents two *tens* (place value) and the face value 5 stands for five *ones*. This knowledge is also key for being able to translate a structured number representation of a two-digit-number (and beyond), in this task by using (pictorial representations of) Dienes blocks, into a symbolic representation and vice versa.

What kind of errors and other warning signals can be expected with this task?
Children who do not yet understand the difference between tens and ones might just count the objects presented and for the first number (25) would arrive at 7, i.e. 2 rods and 5 cubes, 7 objects altogether. Another possible mistake is not paying attention to the order in which the numerals are to be written. For example, for the second number (36), children might not pay attention to the place value and rather write the number in the order of representation, i.e. 6 ones and 3 tens: 63. Errors can also occur due to miscounting, e.g. a child could arrive at 44 for the third number (45) because of miscounting fourteen instead of fifteen ones. Another possible mistake is to write 315 for the third number (3 for the tens, 15 for the units, without considering that 10 units make another ten).

Please note that the ones in the illustrations are deliberately arranged in such a way that children do not have to count them, if they recognize the underlying structure and have acquired the relevant numerical knowledge (e.g., 3+3=6 in the second number). The time limit is deliberately set so tightly that children who count all the ones will probably not finish and/or make counting errors. However, it is in the children's interest if the screening in this way provides evidence that they do not yet know or at least not use such number structures for quasi-simultaneous comprehension. This is especially true for understanding that 5+5=10 and that 10 units make a ten, which is necessary to quickly recognize the third number as 45.

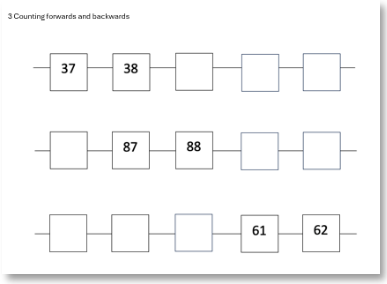
What kind of support could be given to children who show deficits with this task?
This task uses pictures of ten-rods, and cubes for the ones. If this material has not been used in class, difficulties with this task could simply be due to this. In this case, you might better use a different task using material that your students are acquainted with to assess the key skill of interest here.

In any case, it is important to work with children on the understanding of the principles of grouping/bundling and place value using *material*. With regard to the type of materials that are suitable for this purpose, activities should first be set in which the children themselves have to form tens out of ten objects each and learn to take these tens into account when determining and noting down the total number of objects.

In the further, pre-structured ten-rods and one-cubes are useful material for expanding and consolidating understanding. Children should explore the number of cubes in a rod (always 10) and use this knowledge for activities such as replacing groups of 10 individual cubes with one rod ("exchanging") to deal with large quantities of cubes.

Children who do not pay attention to the order of the numerals and confuse numbers such as 34 und 43 need to see and understand the difference between their cardinal representations. This can be achieved by highlighting the place value (tens/ones) of the two face values 3 and 4 to understand that 34 means 3 tens and 4 ones, while 43 stands for 4 tens and 3 ones.

Task 3: Forwards/backwards in the number sequence



Key skill tested with this task:
Continuing the series of natural numbers in both directions starting from a two-digit number, going beyond and below pure tens.

Why is this skill a key skill?
Mastering the number word series is essential for counting forward and backward. Counting forward is particularly important in everyday life. The starting numbers for this task have been chosen so that pure tens must be counted beyond or below. Therefore, errors in this task may indicate a lack of understanding of the decimal place value system (bundling and unbundling). Conversely, however, being able to count does not necessarily mean that the child has a sound understanding of the principle of bundling. Task 3 therefore only covers one aspect of what is important as a prerequisite for further learning. For more information on the importance of understanding place value, see the commentary on Task 2.

What kind of errors and other warning signals can be expected with this task?
Errors such as 39-30-31 (row 1) or 89-80 (row 2) suggest an insufficient understanding of the bundling principle. Starting in row 3 with the wrong ten (e.g., 78-79-60) indicates issues with unbundling.

In row two, in the first field a child could insert the number that s/he would have meant to continue the row on the right if there had been another empty field. In this case, the child might arrive at 91-87-88-89-90, and the problem would not lie in mastering the bundling but in understanding the task format. This can be clarified by talking to the child.

Similarly, if a child in row three continues to the left with 63-64-65, it may be simply because they misunderstood the task. However, it is also possible that the child is overwhelmed by the task of continuing the row in reverse order and is therefore doing what comes more easily. In such cases, further clarification must also be sought through face-to-face talk. Another type of possible error refers to problems with the spelling of two-digit numbers, for example if row 1 is continued with 84 (see Task 4).

What kind of support could be given to children who show deficits with this task?
If errors occur on this task, first check the child's counting skills verbally. These skills cannot be directly assessed with a paper-and-pencil screening. It is particularly important to determine whether the child can count fluently and confidently forward and backward from any two-digit number and whether he or she can also go beyond and below pure tens without any problems.

However, difficulties in this area should not be countered with purely verbal counting exercises. If a child persistently has problems counting over and under tens, he or she probably lacks an understanding of the bundling principle: each 10 ones is a ten, which is why 39 in the series of natural numbers is followed by 40, 89 by 90 and so forth. Bundling activities with materials help to develop this understanding (see Task 2).

Analog difficulties are even more frequent when unbundling, and thus, when going below pure tens. This makes appropriate material actions even more important. To take one unit away from 70, which is represented by seven ten-rods, one ten-rod must be "unbundled" (exchanged) into ten ones. This is why 70 - 1 = 69 and why 69 is in the row of numbers before 70.

Task 4: Writing two-digit numbers

Key skill tested with this task:
Writing number words that are heard, using numerical notation

"Thirty-four" → 34

Why is this skill a *key* skill?
If children repeatedly make mistakes or are unsure when translating between the spoken and written form of two-digit numbers, this makes it much more difficult for them to participate in math lessons, as well as to deal with two- and multi-digit numbers in everyday life. This goes beyond writing and reading numbers in numerical notation: even when doing mental arithmetic, numbers written in numerals are internally translated into number words, and mentally calculated numbers must be written down in numerical notation.

What kind of errors and other warning signals can be expected with this task?
The construction of number words for two-digit numbers is a challenge for children in all European languages. We take here as an example the German language: The German names for the numbers from 10 to 19 follow different rules than those from 20 to 99. From "thirteen - dreizehn" onwards, there is a "twist" in that the place value written on the left (first in the direction of writing) is pronounced after the value of the ones digit written on the right. A common mistake is therefore the "swapping of digits", e.g. when "thirty-four - vierunddreißig" is written as 43. Mix-ups also occur with 15 – "fünfzehn" (could be written 50) and 50 – "fünfzig" (could be written 15).

If a teacher in German speaking countries notices that a child writes the ones digit first when writing two-digit numbers and only then, but to the left of it and therefore with the correct result, the tens digit: this should also be taken as a warning signal. In the long term, it is not a good idea to write numbers in this way.

What kind of support could be given to children who show deficits with this task?
If a child persistently makes mistakes or shows uncertainty in this area, their understanding of the position and bundling principles should be checked first. It only makes sense to target the understanding of number word formation and, based on that, to practice reading and writing two-digit numbers once children understand what tens are (ten ones grouped together form a new unit; bundling principle) and that the position of a digit provides information about whether it is a ten or a one (positional principle).

Even children who understand the bundling and position principles may have difficulty with the peculiarities of number word formation (see above). This is especially true for children with a different mother tongue. For instance, in German speaking schools teachers should keep in mind that in most other languages, the number words are not distorted from 21 onward.

The first step in providing assistance is to thoroughly and repeatedly discuss the rules and exceptions of number word formation in class.

Subsequently, regular "listening training" is recommended. The teacher pronounces a number word, and the children should concentrate on hearing the number of tens only. Alternatively, the children concentrate on the number of ones in the one-place of the given number. In each case, the children should not write down the whole number, but listen carefully, reflecting on place value. Writing from dictation should only be practiced later. Calculator dictations have also proven successful. When children type in numbers they have heard, the calculator forces them to enter the tens first and therefore to analyze the heard number for tens and ones.

Task 5: Halving two-digit numbers

Half of 12: ______

Half of 16: ______

Half of 60: ______

Half of 80: ______

Half of 50: ______

Key skill tested with this task:
Halving two-digit numbers, including pure tens with an odd number of tens.

Why is this skill a *key* skill?
Halving (like doubling) is a basic arithmetic operation in itself. Doubling and halving should therefore be worked on from the first grade, initially in the number range up to 10 and 20, and also *automated* as early as possible. Being able to halve quickly and confidently beyond 20 is the basis for the flexible handling of two-digit (and later, using analogous strategies, multi-digit) numbers.

When working on simple multiplication using core tasks (see Task 12), it is important that children can quickly and confidently divide tens in half. For example, to derive 5 x 7 from 10 x 7, a child must know that 35 is half of 70. Halving is also extremely helpful when dividing. For instance, 48 ÷ 4 can be solved by halving twice (48 ÷ 2 = 24, 24 ÷ 2 = 12).

Halving in the difficulty level of this task is a basic operation that should be (almost) automatic by the end of the 2nd school year. Hence the time limit for it in the screening. When conducting the screening, it is of course important that the children do not feel stressed. Recommendations for implementation can be found in the Manual.

What kind of errors and other warning signals can be expected with this task?
Errors when halving two-digit numbers often indicate problems with the decimal system: children who, for example, do not think of 16 as 10+6, but as “a 1 and a 6”, may then determine 13 as half (6 is halved correctly, but they do not know how to deal with the 1 and write it down again), or even only 3 (the 1 in the tens place is ignored). In an analogous way of thinking, a child can determine 11 as half of 12.

For children who use (supposedly) memorized doublings in the number range up to 20, the error could be, for example, 9 as half of 16 (incorrect memorization 9+9=16) or 7 as half of 12 (incorrect memorization 7+7=12).

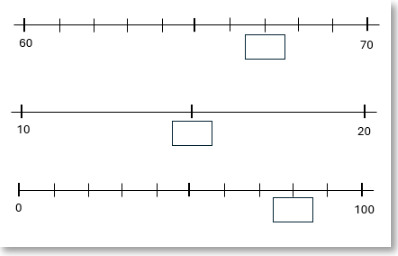
Some children avoid writing down the answer if they don't know it. Some children, if asked about 50, might explain that 50 (such as 30, 70, 90) can't be halved. This suggests they see 50 as "five-zero," not five tens. Others might write 20 or 30 as the answer, thinking of 2+3 as the one splitting of 5 that comes nearest to two equal parts. Errors in recall can also occur when halving 60 and 80 (40 as half of 60, conversely 30 as half of 80).

Errors of any kind should be investigated by talking to the child. If it is more than a careless mistake, it will occur repeatedly, and the source of the error can then be identified in conversation.

What kind of support could be given to children who show deficits with this task?
As an important basic operation (see above), halving must be worked on carefully and with the help of material long before the more comprehensive operation “division” becomes a topic. Halving the numbers up to 20 should be worked on together with doubling the numbers up to 10, with halving as a reversal of the corresponding doublings. When doubling the numbers from 6 to 9, working with the “power of five” strategy proves its worth: to double 8, for example, 8 can be thought of as 5+3. 5+5=10, 3+3=6, the double of 8 is therefore 16. Half of 16 is, conversely, 8. The finger representations of 8 (two children working together), used non-counting, are suitable for working out the power of five, as well as corresponding representations in the 20 frame, which are interpreted according to the power of five.

Halving pure tens, such as 30, 50, 70, and 90, is an important activity for gaining basic understanding of the place value system. For instance, to halve 50, one must unbundle a ten. Children should be asked to represent 50 (or 30, 70, or 90) with 5 (or 3, 7, 9) ten-rods or ten-euro notes of play money and figure out how to divide the number into two equal portions. If they can use the material, even children with learning difficulties will often discover for themselves that in such cases one of the tens must be exchanged for ten ones. However, it is crucial that this subsequently becomes solvable by them using mental representations, and, in the long term, even automated. Working on halving in this way also helps consolidate understanding of the decimal system.

Task 6: Numbers on number lines



Key skill tested with this task:
Allocate the appropriate two-digit numbers to given markings on a number line, taking into account the different scales of the number lines used.

Why is this skill a *key* skill?
Number representations on number lines are important tools in mathematics lessons from primary to secondary school. Numbers in any number range and not only natural numbers can be represented on number lines with minimal effort. Representations on number lines can help to clarify and understand relationships between numbers and operations with numbers. However, this requires a sound interpretation of such representations.

Task 6 examines an important aspect of such interpretations, namely the observation of different scales: The distance between two adjacent markings on the first number line represents one, on the second five and on the third ten. To correctly determine the markings shown, children must pay attention to the labeled edge markings as well as the number of equally long distances between the edge markings.

In addition to skills related to the means of representation itself, the task provides information on whether children can halve (e.g., 5 as half of 10 on number line 2) and on their ability to work with two-digit numbers. On number line 1, depending on their strategy, children will either continue counting from 60 or recognize the middle mark between 60 and 70 as 65 and continue counting from there. On number line 3, they could count in steps of ten, starting from 0 but also from 50, if they use the center mark. Alternatively, they could recognize that each distance is ten units long and that two tens are missing from the displayed mark to 100.

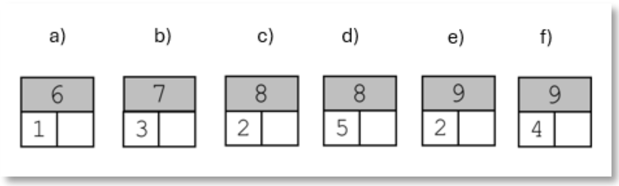
What kind of errors and other warning signals can be expected with this task?
Errors in counting can occur on number line 1 (e.g., 66 or 68 instead of the correct 67). If a child enters 76, it is likely due to problems with writing numbers (see Task 4). In combination with a counting error, such problems (confusing the numerals) can lead to writing 86 instead of 68. If a child enters 7, ignoring the tens, this mistake, like others, should not be hastily dismissed as carelessness. Only a conversation in which the child is invited to explain their thought process can clarify the reason behind a mistake.

The same applies if 5 is given as the middle number on number line 2 instead of 15. Errors such as 14, 16, or 17 suggest that the child counted on from 10 and imagined, or actually made on the test sheet, markings at self-chosen (estimated) intervals. If a child has written 11 or 19, they probably counted forward or backward by one from the left or right edge number, paying attention neither to the scale of fives nor to the proportionality required on the number line.

Similarly, a child may arrive at the entry 98 on number line 3. Other possible errors are 40, when counting in steps of five, and starting with 0, or 90, when counting in steps of five backwards starting from 100. As explained, clarity about how these and other errors come about requires an individual dialogue with the child in question.

What kind of support could be given to children who show deficits with this task?
Many textbooks for grade 2 feature number lines with markings primarily or even solely at one-unit intervals. Moreover, some textbook-authors seem to assume that number line representations speak for themselves. However, it is important to specifically teach children how to interpret number lines not only in terms of counting, but also and more importantly in terms of *measurement*. For example, they should understand that the mark 8 on a number line between 0 and 10 indicates that the distance from 0 to 8 is *eight units long*. This distance can be divided into five and three units or halved into two sections of four units each, and so on. Subsequently, children should learn to recognize *distances of ten* each within number lines up to 100 and understand that number lines can be scaled differently. Depending on the specified distance that has been chosen to represent the ones (or the tens) on a given number line, the distances for two, three, and so on up to ten ones (or two, three, and so on up to ten tens) are multiples of this unit distance.

Task 7: Splitting numbers



Key skill tested with this task:
Basic number facts (splitting numbers up to 10)

Why is this skill a *key* skill?
For the further development of arithmetic skills, it is fundamental that children learn to understand natural numbers (initially up to 10) as compositions of numbers ("part-whole concept") and automate all ways in which numbers up to 10 can be composed out of or decomposed into two numbers. For example, if a child automatically associates the number eight with 2+6 (subtask c) and 5+3 (subtask d), they will be able to solve problems such as 2+6, 8-6, 8-5, 3+5, 3+_=8, and so forth, without having to count. They can easily break down a task such as 37+8 into two simpler steps: 37+3, 40+5. Likewise, they can break down a task such as 32-8 into 32-2, 30-6. Splitting numbers up to 10 should be automated by the end of the first school year, hence the time limit in the screening. To avoid stress for the children, see the recommendations in the Manual.

What kind of errors and other warning signals can be expected with this task?
Errors could indicate incorrect memorization, for example if a child enters the number 4 in subtask 1 or the number 5 in subtask 2. Especially in such cases, where the correct number is missed by one, a counting error could also be behind it. The child would then not have recalled an incorrect number from memory but would have tried to solve the decomposition by counting and (typically) miscounted by one.

Errors such as inserting 7 in subtask a) suggest that the child has misunderstood the task and calculated the *sum* of the two given numbers. If you have never practiced number decompositions in your class in the format used here, such errors should not be taken as a valid statement about the child's number decomposition skills. It would then be better to check the number splitting asked for here in another written or verbal form. However, it is also possible that such errors are the result of automatisms that are not accompanied by understanding. One-to-one discussions can shed light on this.

What kind of support could be given to children who show deficits with this task?
If children have not mastered splitting numbers up to 10 by the end of their first year of school, they will likely experience significant difficulties in the future. In such cases, an individual discussion must first clarify whether the problems lie in the area of automation or if there is a lack of the fundamental understanding that numbers are made up of numbers (parts-whole concept).

Activities involving structured number representations help develop an understanding of parts and wholes. Depending on the representations used, to be able to identify the parts, children must be able to recognize numbers up to four at a glance without counting. However, it is known that some children fail to subitize even three. In these cases, strategies must be developed to compensate for this, such as presenting the number three within a frame of five. This allows for quasi-simultaneous recognition of three via the two empty fields, which even children with limited abilities in simultaneous recognition can usually perceive without counting. Fingers also offer good opportunities to make part-whole relationships up to ten accessible to children with learning difficulties, provided that children learn to use them for representing numbers without counting. Finger representations as well as dot representations in frames of fives, tens and later twenties can be used for flash exercises: Numbers presented in a structured way should be grasped “at a glance”. The decompositions that make this non-counting comprehension possible should be verbalized by the children and also used for non-counting operations. E.g., if a child records eight as five and three, they should think about what happens if five is taken away, etc.

For children with learning difficulties, it is a good idea to initially focus on a few types of decompositions (power of five, decomposition into two halves) and reinforce these. Subsequently, children should work on understanding how single decompositions of a number connect, according to the principle of compensation: e.g., eight is five and three, but also six and two; the one part increases by one, the other decreases by one to compensate. It is only based on such insights that in the next phase also trying to *automate* the basic number decompositions becomes promising. Using flashcards has proven to be an effective way to do so.

Task 8: Addition

Key skill tested with this task
Adding in the number range up to 100, including bridging tens.

32 + 7 =

6 + 74 =

60 + 30 =

27 + 40 =

25 + 8 =

Why is this skill a *key* skill?
The ability to quickly and accurately add in your head is a fundamental mathematical skill of far-reaching importance, even in everyday life. This remains true in the digital age, where calculators are easily accessible. If you are not able to do mental math, you won't catch incorrect entries. In the hierarchy of school mathematics, adding up to 100 is a prerequisite for adding in higher number ranges. It is also necessary to derive multiplication facts from other, already memorized facts (e.g., 6 x 7 from 5 x 7 by adding 35 + 7). Also multiplying two-digit and multi-digit numbers requires adding partial results, and so on.

It is important that children learn to solve addition problems *without counting,* and do so as early as in the first year of school. The screening cannot record which strategies (counting or non-counting) children use to solve arithmetic problems. However, as far as this is possible in the classroom, pay attention to whether children are openly counting when working on this task or whether they give covert signs of counting (staring upwards, nodding their heads...). To avoid stress, no time limit is set for this task in the screening. However, children who use counting strategies usually take considerably longer to reach the result. If you observe this, it provides important additional information about whether individual children will need further support to finally develop non-counting calculation strategies.

What kind of errors and other warning signals can be expected with this task?
Children who count (see above) tend to make more mistakes when adding than other. Typical are "errors by one" such as 32+7=38, because 32 is counted as the first number when counting on. Other errors can be explained by a misinterpretation of the fingers used for counting. For example, a child who first holds up five fingers for 25+8 and then wants to add eight fingers one after the other, could first fill the second hand and then continue with the first hand, clenching it into a fist and stretching out three more fingers. When they then still hold up also the five fingers of the second hand, they see eight fingers and could mistakenly interpret this as 38.

A second source of error involves problems with ones and tens. For example, in the case of 6+74, a swap of digits (47 instead of 74) can lead to the incorrect result of 53. In the case of 27+40=76, the swap of digits probably occurred when writing down the result (76 instead of 67). Adding 6 and 4 without considering the bundling of a further ten necessary for 6+4 leads to the result 6+74=70. In the case of 27+40, adding (or counting on) 4 instead of 40 leads to the result 31. Another possible error with 6+74 is 134, if children add first 6+7=13 and then write a 4 after the 13. Many errors can only be clarified through individual discussions, and not always with complete certainty. However, it is always worth trying to understand them, as there is often a pattern behind errors and, once the pattern is understood, the child can be helped more effectively.

What kind of support could be given to children who show deficits with this task?
Support activities in this area are only effective if they address the root of the child's arithmetic difficulties. If children add by counting, this usually involves an insufficiently consolidated understanding of part-whole thinking in the number range of 10 and a lack of available automated part-whole relationships (see Task 7). If children still demonstrate deficits in this area by the end of their second year of school and beyond, fundamental remedial activities are urgently needed. Such cases often require individual support beyond what is possible in the classroom. The same applies if deficits in understanding the decimal system are also, or primarily, responsible for the problems with adding.

Even if working through such fundamental difficulties at higher levels of elementary school is difficult and requires a lot of perseverance and patience from both sides, the child and the teacher, it is the only way to prevent the child from facing even greater difficulties from year to year. Using aids to help the child with adding by counting (e.g. by handing out a 100 board), on the other hand, would be counterproductive.

Task 9: Subtraction

48 – 6 =

37 – 7 =

20 – 9 =

56 – 30 =

25 – 8 =

Key skill tested with this task
Subtraction in the number range up to 100, including bridging tens.

Why is this skill a *key* skill?
What has already been said about Task 8 (Addition) largely applies here as well.

What kind of errors and other warning signals can be expected with this task?
As with Task 8 (Addition), errors in Task 9 are often counting errors ("off by one") or misinterpretations of the fingers used for counting. It might also be that a child has not used counting for subtraction but made an error due to an incorrect memorization of a basic fact (e.g., 8-6=3).

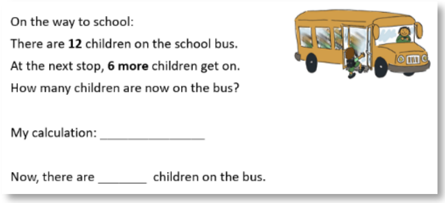
Other errors involve problems with the decimal system, such as swapping digits, incorrectly linking tens and ones, and failing to take unbundling into account in tasks with bridging tens.

Finally, errors may indicate a combination of both the aforementioned problem areas.

What kind of support could be given to children who show deficits with this task?
As mentioned earlier, all that has been already commented to Task 8 (Addition) largely applies here as well.

Studies show that difficulties with subtraction are even more common than with addition. However, from a mathematics education perspective, this is not because subtracting is *objectively* more difficult than adding. Of course, it is more difficult when *counting* is used as a strategy, because counting backwards has generally been practiced less and is therefore more error prone. If also children who do not subtract by counting find subtraction more difficult and make more mistakes than when adding, at least part of the explanation could be that subtraction was covered and practiced less intensively in class. If the screening for your class shows clear differences in the children's performance in addition and subtraction (and then presumably to the detriment of subtraction), this should in any case be an opportunity to review the weighting of the two basic operations in lessons.

Task 10: Word problem 1 (addition)



Key skill tested with this task
Solving a text problem that can be solved in one calculation step by the appropriate addition ("simplex problem").

Why is this skill a *key* skill?
Text problems of the type presented here (simplex problems) are the elementary form of real-world arithmetic problems (presented in text form). Simplex problems provide an indication of whether a child has built up and can recall basic ideas of an arithmetic operation. This is an essential prerequisite for being able to solve more complex real-world problems. The type of basic concept of addition that is assessed by Task 10 is *addition as joining*. This concept is often built up already at kindergarten age. Other basic concepts that are also desirable (e.g. *additive comparison*) are not assessed in the screening.

What kind of errors and other warning signals can be expected with this task?
In this task, children should not only write down the resulting number, but also the calculation by which they arrive at this number. If the calculation is not written down, it is possible that the child has not noticed or ignored this request; and if the result is correct, it can be assumed that 12+6 has been calculated. However, it cannot be ruled out that a child understands the context and arrives at a solution by counting but does not realize that the solution can be written down symbolically as addition. This is considered in the evaluation insofar as a full point is only awarded if both the calculation and the result are written down correctly. In such cases, however, it should be clarified in discussion with the child whether s/he really does not know how to assign the fitting addition term to a text task.

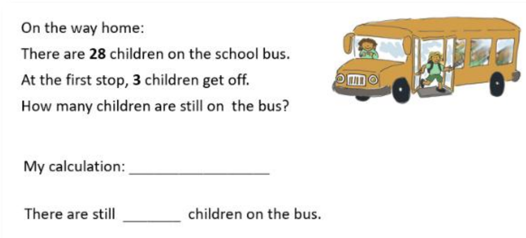
If no calculation was written down, even if the result is wrong, the child could nevertheless have calculated 12+6, but with a calculation error. If the child has calculated by counting, also for this task the correct result might be missed by 1 (12+6=17 or 12+6=19) due to a counting error (see Task 8). 12+6=8 could arise if the 1 in the tens place is ignored. This should not be hastily dismissed as a careless mistake. Through discussion and by looking at other tasks, it can be clarified whether the child has fundamental problems in dealing with two-digit numbers.

If the child writes down 12-6 as a calculation, you should first check how the same child solved the following Task 11 (text problem for subtraction). In a one-to-one conversation, further simplex problems for addition and subtraction should then be used to confirm the actual level of competences.

It should be noted that the task is presented verbally. The text is only offered additionally. This is intended to prevent possible problems with reading comprehension from overshadowing the assessment of the mathematical skills of interest here. Nevertheless, in the case of children whose reading skills are known to be limited, it should be considered that it may have an influence on their performance in this and the other text tasks read aloud (11, 14, 15) that for them the text is not available in addition to what they hear. Since children (especially those with reading difficulties) must listen attentively during this task, attention difficulties (general or only selective distraction during this task) can play an even greater role than with other tasks.

What kind of support could be given to children who show deficits with this task?
For children to develop a solid basic understanding of the four basic arithmetic operations, it is crucial that they can relate them to experiences they have had and continue to have in everyday life. In this sense, the operation signs should be related to actions and real-world situations from the very beginning. Important tasks and exercises include translating between an arithmetic term such as 3+6, actions with materials, real-world situations (also presented as a text, as in this example) and drawings. It is essential that this translation is demanded in both directions, i.e. also in such a way that children should carry out suitable actions, invent word problems and create drawings themselves that are fitting to a given term, and explain in as far their actions, word problems, and drawings fit to the term. When working with word problems, it should be repeatedly discussed beyond solving the individual problem what is typical in each case, e.g. of problems that can be solved by addition.

Task 11: Word problem 2 (subtraction)



Key skill tested with this task
Solving a text problem that can be solved in one calculation step using the appropriate subtraction (“simplex problem”).

Why is this skill a *key* skill?
What was explained for Task 10 is largely analogous. Task 11 addresses the most basic idea of subtraction, which is *"taking away."* Children usually develop this concept in the first year of school at the latest. Other important concepts not covered here include *"comparing"* and *"missing addend".*

What kind of errors and other warning signals can be expected with this task?
As with Task 10, when evaluating the screening, it is important to pay attention to possible calculation errors on the one side and errors in the choice of calculation on the other. The latter are evident when the child writes down the calculation as requested. If the calculation written down is not 28-3, what has already been stated for Task 10 applies also here, analogously.

If the child only notes the result, a correct result (i.e. 25), as in Task 10, suggests that the child is aware that the question described in the text can be “mathematized” by writing 28-3. However, this should also be confirmed in discussion with the child. Talking to the child will also clarify the underlying problem if in the screening an incorrect result, due to its proximity to the correct one, suggests that the child has calculated 28-3, but has made a calculation or counting error and has therefore written down maybe 26 as the solution. The possibility that place values are not taken into account should also be noted here, and only a discussion can clarify whether the resulting errors (for instance 5) indicate more fundamental problems with the decimal system or can be classified as carelessness and concentration errors.

Solutions such as 31 or (in the case of an additional calculation error) 30 indicate that the child has added rather than subtracted.

What kind of support could be given to children who show deficits with this task?
See what has already been commented on Task 10. Note that, although the basic idea of subtraction as taking away, which is addressed in Task 11, is fundamental, it is important to work intensively and persistently with children on other basic ideas once this concept has been mastered. Ideally, children will learn in their first year of school that by subtraction they can also determine the difference between two numbers or calculate how much is missing from the given whole when they know one part of it. If the screening does not indicate any problems with the concept of taking away, that is, of course, pleasing. However, more far-reaching goals should be set and pursued in lessons. It is also important to *repeatedly* check whether children have sustainable, diverse and varied concepts of subtraction, as with all four basic arithmetic operations.

Task 12: Number facts (multiplication)

a) 7 × 2 =
b) 4 × 5 =
c) 8 × 10 =
d) 9 × 2 =
e) 10 × 7 =
f) 5 × 6 =

Key skill tested with this task
Quick and accurate solving of the core tasks (with factors 2, 5 and 10) within the multiplication table.

Why is this skill a *key* skill?
The multiplication table is part of the basic arithmetic facts. Children should master *all* multiplications with factors up to 10 before tackling more complex tasks, such as multiplying two-digit numbers, dividing, and doing arithmetic with fractions.

The point at which *all* the tasks of the multiplication table should be mastered depends on the math lesson. In Screening 2+, we deliberately assess only tasks with the factors 2, 5 and 10. These can be regarded as *core tasks* of the multiplication table. Current didactic concepts recommend focusing on these core tasks in an initial phase of automatic practice. Children can and should use these tasks to first derive the other tasks of the multiplication table. This subsequently makes it easier to automate also these other tasks. Mastering the core tasks of the multiplication table, the subject of Task 12, can therefore be regarded as a key competence required for developing the more advanced key competence of "mastering the whole multiplication table". Note that the time limit set for Task 12 is important in order to get at least indications whether the child has really *mastered* the core tasks (see below for further explanations).

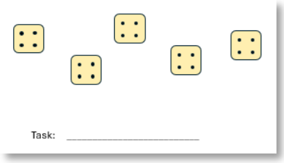
What kind of errors and other warning signals can be expected with this task?
A basic arithmetic fact can be considered automated if the task is solved reliably and quickly without further thought, either by direct retrieval from long-term memory or by fast, quasi-automatic deduction (e.g. if a child does not spontaneously think of 18 for 9 x 2, but first thinks of 2 x 9 in order to arrive at the correct result by swapping the factors with a minimal time delay). Please note that a paper-and-pencil screening cannot provide reliable information about whether tasks are automated in this sense. In the literature, the time limit for "basic fact mastery" is usually set at a maximum of 3 seconds. However, it should be considered that the children must first read the tasks and then write the answers. Hence the time limit of 30 seconds for 6 tasks. To solve all 6 tasks correctly in this time should usually be no problem for children who have automated them. However, a child may not succeed in solving all the tasks in time because he or she works more slowly overall, is distracted, has difficulty in writing, and the like. On the other hand, it is possible that a child has indeed not automated *all* six tasks but can still solve them in 30 seconds. For instance, he or she could solve some by recalling them from memory and others by quickly counting within the series. This strategy is not effective in the long term. Task 12 is therefore an *attempt* to find out whether children have mastered some of the core tasks of the multiplication tables, no more, no less. Even as an attempt, it only works if the time limit is observed when carrying it out. In the manual you will find tips on how you can do this in such a way that the children are not stressed and children who do not complete all the tasks in the given time are not frustrated.

In addition to omissions due to lack of time, there are two main types of errors: a) Recall errors occur when the child spontaneously remembers an incorrect result ("misremembers"). These errors often involve results of other multiplication tasks, such as 5 x 6 = 54, 2 x 9 = 40, and 10 x 7 = 27. b.) Errors such as 5 x 6 = 25 can probably be explained by the child counting by fives and either taking one too few or one too many steps, as in 5 x 6 = 35.

What kind of support could be given to children who show deficits with this task?
As explained, current didactic approaches recommend that the multiplication tables should not be worked on in isolated series of tables (e.g., the ten tasks from 1 x 4 to 10 x 4 forming one table, the tasks from 1 x 6 to 10 x 6 forming another, and so on), nor should the focus be on pure memorization. Rather, an attempt should be made to first secure the core tasks with factors 2, 5 and 10, and then to develop targeted derivation strategies with the help of which the children can derive all further tasks from the core tasks (such as 9 x 7 from 10 x 7, or 6 x 8 from 5 x 8, etc.). The targeted automation of all non-core tasks is subsequently facilitated by the fact that the children can use the already memorized core tasks together with relational understanding of derivation strategies as memory anchors. Basis for this is a sound conceptual understanding of multiplication (see Task 13).

Task 13: Interpreting a representation as multiplication

Key skill tested with this task
Interpreting a representation of several equal quantities as a representation of a multiplication.



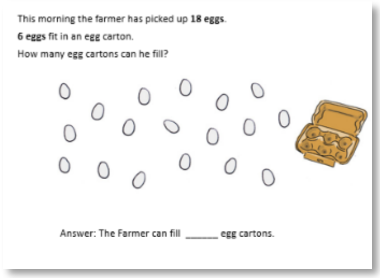
Why is this skill a *key* skill?
Task 13 addresses the key competence of "fundamental concepts of multiplication." Such fundamental concepts are prerequisites for developing internal mathematical skills, such as understanding division as an inverse operation, recognizing and using multiplicative relationships between numbers, and developing proportional thinking and elementary algebra. They are also prerequisites for recognizing multiplicative relationships in real-life situations and problems, and for using multiplication (even with the help of a calculator) to solve them. The fundamental ideas about basic arithmetic operations involve many dimensions. Task 13 covers only one of these dimensions, and even then, only to the extent that a paper-and-pencil screening allows. This dimension is the ability to assign a drawing to a suitable arithmetic term.

What kind of errors and other warning signals can be expected with this task?
One possible error is to write 4 × 4 × 4 × 4 × 4. If a child writes down 4+4+4+4+4 as the answer, they have written down a fitting addition but have not followed the demand to write down a fitting multiplication. This may indicate that they do not actually associate images of this kind with multiplication. This does not necessarily mean that they have not gained an understanding of multiplication as an independent operation, but it should certainly be considered a warning sign. The same way of thinking, combined with counting errors, can also result in terms such as 4+4+4+4 or 4+4+4+4+4+4.

In different countries there are different conventions regarding the way multiplication is introduced in textbooks. In German-speaking countries, for example, multiplication is introduced as an abbreviated notation for the addition of equal summands based on the principle of "how many times a certain number/quantity." According to this convention, only 5 × 4 is suitable for representing five dice, each showing four dots, while 4 × 5 would be fitting to a drawing of four dice, each showing five dots. In Italy, as a counterexample, 5 x 4 is introduced as abbreviation of 5+5+5+5. However, regardless of how you introduce multiplication in class, studies show that you must expect some children to think differently from what they have heard and seen in their lessons. Therefore, also a German student might think of the five dice with four dots each shown in the illustration as "four times five" and therefore write down the term 4 × 5, thinking of it as "a four that can be seen five times." Such individual interpretations that deviate from convention can cause problems in the classroom because there is a risk of misunderstandings arising as a result. However, it would not be appropriate to consider them wrong; children who write down 4 × 5 also demonstrate an understanding of multiplication.

What kind of support could be given to children who show deficits with this task?
As already commented on Task 10, it is important for children to be able to relate basic arithmetic operations to everyday experiences for developing a solid conceptual understanding. In this way, they should initially associate multiplication tasks with actions and situations in which the same number/quantity has to be taken repeatedly or thought of together. It is important to highlight both the similarities and the differences compared to addition. Tasks such as creating representations and inventing word problems that match a given term, or conversely, asking children to write down the appropriate term for a representation or a word problem, are helpful in this regard. At this early stage, commutative terms such as 5 × 4 and 4 × 5 should *not* be treated as equivalent. Developing an understanding of the interchangeability of factors is an important *next* step. First, however, children must learn to distinguish mentally between the number that answers the question "How many times?" and the number that indicates "How much each time?" Hereby, to facilitate communication in the classroom, it is useful to follow the convention set by national textbooks. Children who nevertheless interpret things differently should be asked to explain their interpretation. However, the same applies to all children: Visual representations or illustrations using materials should never be taken for granted. They only contribute to the development of sound basic concepts if they are used to clarify mathematical ways of thinking in conversation.

Task 14: Word problem 3 (quotitive problem)



Key skill tested with this task
Solving a (read aloud) word problem in which a total quantity must be divided into subsets of specified equal sizes.

Why is this skill a *key* skill?
Task 14 can be solved mathematically as a division problem: 18:6=3. Here, 6 corresponds to the size of one of the equal portions into which the total number 18 is divided, and the result 3 corresponds to the number of these portions. In literature, this is referred to as “quotitive division,” as opposed to “partitive division” (see Task 15), the second important basic concept of division that children should develop and consolidate in elementary school. A solid understanding of both types of division is a prerequisite for both the next steps in the hierarchy of school mathematics (e.g., understanding fractions, dividing rational numbers…) and for solving more complex word problems.

Screening 2+ deliberately does not check whether children solve the task using division. Whether they do so, depends on whether and to what extent division has already been covered in class, especially in connection with word problems. Task 14 only assesses whether children can understand the situation described in the text and solve the problem it contains. They may use the drawing, draw in it, try things out, and do not think about division at all.

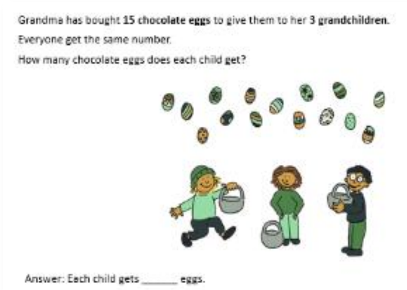
What kind of errors and other warning signals can be expected with this task?
The solution 12 (or 13 or 11 in the case of counting errors) can probably be explained by the fact that the child calculated (or counted) 18-6. They might also add 18 and 6; if the addition is correct, they arrive at 24; if there are counting errors, it could also be 25 or 23. If a child used the illustration as a solution aid, an incorrect solution usually also provides clues as to what led to the error.

What kind of support could be given to children who show deficits with this task?
As already commented on Tasks 10, 11, and 13 with regard to the other three basic arithmetic operations, the development of sound basic concepts requires the child to connect the operation to everyday experiences. In the case of division, it is important to draw on experiences in the areas of quotitive division problems (task 14) as well as in partitive problems (task 15). Everyday quotitive problems include: packaging tasks such as the one formulated here; grouping tasks (how many teams of 6 children can be formed if there are 18 children in the class?); measuring tasks (how many 2-liter jugs can be filled with 10 liters of juice?); money tasks such as "How many ice cream scoops costing 2 euros each can I get if I have 8 euros to spend on ice cream?”

The division symbol should be introduced in connection with solving such problems (with the aid of materials), namely as a practical symbol for noting down problems of this kind. Once again, translations in the other direction are also important, i.e., inventing word problems for given terms such as 12:4. Children should also translate such terms into fitting material actions, make fitting drawings, and conversely learn to interpret drawings such as 3 boxes of 6 eggs each as (the result of) division. From the outset, problems involving remainders should also be considered, together with reflecting about whether and to what extent the remainder plays a role in finding the correct number that is asked for as the solution of such problems.

In any case, it is crucial that children learn to understand division as an independent arithmetic operation and not just as “reverse multiplication.” As for the difference between quotitive and partitive division, it is essential that children learn to “mathematize” real-life situations of both types as division and that they associate a given division term with (concepts of) real-life problems in both variants. Technical terms such as “quotitive” and “partitive” do not help them in this regard, but it does help if they can describe the similarities and differences between the two variants in their own language.

Task 15: Word problem 4 (sharing)



Key skill tested with this task
Solving a (read aloud) word problem in which a total quantity must be divided into a specified number of subsets of equal sizes.

Why is this skill a *key* skill?
Like Task 14, Task 15 can be solved mathematically as a division, in this case 15:3=5. Here, 3 corresponds to the number of equal portions into which the total number 15 is divided. The result 5 indicates the size of *one* portion. In literature, this is referred to as “partitive division,” as opposed to “quotitive” (see Task 14), the second important basic concept of division that children should develop and consolidate in elementary school. As already commented on Task 14, a solid understanding of *both* types of division is a prerequisite for both the next steps in the hierarchy of school mathematics (e.g., understanding fractions, dividing rational numbers…) and for solving more complex word problems.
Likewise, it has already been commented on task 14 that Screening 2+ deliberately does *not* check whether children solve the task using division. Whether they do so, depends on whether and to what extent division has already been covered in class, especially in connection with word problems. Task 15 only assesses whether children can understand the situation described in the text and solve the problem it contains, even if they use the drawing, draw in it, try things out, and do not think about division at all.

What kind of errors and other warning signals can be expected with this task?
Partitive problems are more difficult to solve graphically than quotitive problems. In the latter (such as the problem of Taks 14), the goal is achieved by circling subsets of the specified size and counting the resulting subsets. In partitive problems, however, the size of a subset is precisely what needs to be determined graphically. In the example task, this is possible for instance by connecting one egg after the other sequentially to one of the three children after the other, paying attention to “share equally”. The drawn connection lines quickly become confusing. Mistakes are usually due to resulting counting errors, incorrect graphical assignments, and the like.

What kind of support could be given to children who show deficits with this task?
The comments made on Task 14 regarding quotitive problems apply here as well. Note that there are differing positions in current maths education literature as to whether it is more effective to introduce quotitive and partitive division more or less simultaneously or to focus on only one of the two variants at first. However, the majority of maths educators recommend concentrating on one of the two variants when introducing division in class. Quotitive division offers the advantage that problem solving is more accessible with materials and illustrations (see above). In any case, once children have been confronted with problems involving both variants (quotitive and partitive) as two types of division (that is, of making equal portions out of a given number), it is important that children become aware of the similarities and differences between these two types. Keeping this awareness alive requires repeated, targeted teaching activities over a long period of time, but it is important so that children can develop a solid understanding of division and continue to develop this beyond the realm of natural numbers.

4 Notes on the Evaluation and Documentation of results

To help you evaluate the test results, various tools are available for download at https://www.ditom.org/:

If you prefer to evaluate the tests manually, we provide the following aids:

a) An **overview sheet for scoring**, which lists for each task the criteria for awarding one point, half a point, or no points (see page 40);
b) A **class evaluation sheet** for recording and documenting the results of the entire class (see page 41).
c) An **individual evaluation sheet** for recording and documenting the results of a single child, if you wish to keep an individual overview (see page 42);

A much less time-consuming option is to evaluate the results in Excel on your computer. For this purpose, you can download:

d) A **pre-programmed Excel file** with two worksheets that you can switch between via the tabs at the bottom left.

In the sheet titled "qualitative", simply enter, in the appropriate column for each child, the numbers the child wrote in their test booklet as answers to each sub-task. If a child left an item blank, please enter 999.

When you have finished entering the data, switch to the "quantitative" sheet. The program will then automatically indicate whether each sub-task was answered correctly (1) or incorrectly (0) and will calculate the appropriate score for each overall task (1 / 0.5 / 0). At the end of each row, you'll find the percentage of correctly solved tasks and the total score for the individual child. At the end of each column, you'll find the percentage of children in the class who solved that particular task correctly.

The "Critical Score Thresholds" for *DiToM 2+* — and How to Interpret Them

As explained in Section 1, *DiToM* is not intended to label children. Please refer back to the discussion of *DiToM's* goals and guiding principles in that section.

There you will also find a more detailed explanation of the "critical score thresholds," which were determined based on pilot testing of *DiToM* (for version 2+, with 2,020 students across the project's seven partner countries) using the statistical method of Latent Class Analysis. This method makes it possible to assign children, based on their total score in *DiToM 2+,* to one of the following three groups:

Score Range	**Group**
0 to 9	A - Signs of broad difficulties across several key areas
9,5 to 12,5	B - Indications of difficulties in some key areas
13 to 15	C- No indication of major difficulties in key areas

A final note referring back to Section 1: Keep in mind that a screening provides only a snapshot. The results should therefore be compared with your own classroom observations and experiences and, where indicated, used as a starting point for follow-up interviews with individual children — to deepen, refine, or expand your understanding, and, if necessary, to adjust your conclusions at least in part.

Evaluation and Scoring DiToM Screening 2+ (max. 15 points)

1	Counting	1 P. 0 P.	correct quantity (23) all other solutions
2	Tens-ones-representations	1 P. 0,5 P. 0 P.	all three numbers correct (25, 36, 45) two numbers correct all other solutions
3	Forwards and backwards in the number sequence	1 P. 0,5 P. 0 P.	all three rows correct (39,40, 41) (86 .. 89, 90) (58, 59, 60 ...) two rows entirely correct all other solutions
4	Writing two-digit numbers	1 P. 0,5 P. 0 P.	all five numbers correct (34, 15, 43, 50, 67) four numbers correct all other solutions
5	Halving two-digit numbers	1 P. 0,5 P. 0 P.	all five numbers correct (6, 8, 30, 40, 25) four numbers correct all other solutions
6	Numbers on number lines	1 P. 0,5 P. 0 P.	all three numbers correct (67, 15, 80) two numbers correct all other solutions
7	Splitting numbers up to 10	1 P. 0,5 P. 0 P.	all six numbers correct (5, 4, 6, 3, 7, 5) five numbers correct all other solutions
8	Addition	1 P. 0,5 P. 0 P.	all five results correct (39, 80, 90, 67, 33) four results correct all other solutions
9	Subtraction	1 P. 0,5 P. 0 P.	all five results correct (42, 30, 11, 26, 17) four results correct all other solutions
10	Word problem 1 (addition)	1 P. 0,5 P. 0 P.	correct term and result (12 + 6 = 18) either the term OR the result was noted correctly all other solutions
11	Word problem 2 (subtraction)	1 P. 0,5 P. 0 P.	correct term and result (28 – 3 = 25) either the task OR the result was noted correctly all other solutions
12	Core multiplication facts	1 P. 0,5 P. 0 P.	all six results correct (14, 20, 80, 18, 70, 30) four or five results correct all other solutions
13	Interpreting a representation as multiplication	1 P. 0 P.	correct term (5 × 4 or 4 × 5), result is irrelevant all other solutions
14	Word problem 3 (quotitive problem)	1 P. 0,5 P. 0 P.	correct answer (3 egg cartons), drawing is irrelevant bundles of six are circled, but answer „3“ was not noted all other solutions
15	Word problem 3 (sharing)	1 P. 0,5 P. 0 P.	correct answer (5 chocolate eggs), drawing is irrelevant correct drawing, but answer „5“ was not noted all other solutions

Class Overview DiToM Screening 2+ (max. 15 P.)

Class .... Date ....

Child's name	Age Y;M	Individual notes	1 Counting	2 Tens-ones-representations	3 Numbers forward and bac-ward	4 Writing two-digit numbers	5 Halving two-digit numbers	6 Number line	7 Number splitting	8 Additon	9 Subtraction	10 Word problem 1 (addition)	11 Word problem 2 (subtraction)	12 Core multipli-cation facts	13 Representation of multiplication	14 Word problem 3 (quotative)	15 Word problem 4 (sharing)	**Overall score**

DiToM
Diagnostic Tool in Mathematics

Name: ______________________________

Date: ______________________________

Evaluation form DiToM Screening 2+

Item	Right answer	Check right/ wrong	Points
1	23		
2.a	25		
2.b	36		
2.c	45		
3.a	394041		
3.b	868990		
3.c	585960		
4.a	34		
4.b	15		
4.c	43		
4.d	50		
4.e	67		
5.a	6		
5.b	8		
5.c	30		
5.d	40		
5.e	25		
6.a	67		
6.b	15		
6.c	80		
7.a	5		
7.b	4		
7.c	6		
7.d	3		
7.e	7		
7.f	5		

Item	Right answer	Check right/ wrong	Points
8.a	39		
8.b	80		
8.c	90		
8.d	67		
8.e	33		
9.a	42		
9.b	30		
9.c	11		
9.d	26		
9.e	17		
10 part 1	12+6=18		
10 part 2	18		
11 part 1	28-3=25		
11 part 2	25		
12.a	14		
12.b	20		
12.c	80		
12.d	18		
12.e	70		
12.f	30		
13	5*4 or 4*5		
14	3		
15	5		

Total points achieved out of 15

Comment: ______________________________

Valuation:

Items 1 and 13	correct = 1 point; incorrect or missing = 0 points
Items 2, 3 and 6	all 3 correct = 1 point; 2 correct = 0.5 points; 1,0 correct or missing = 0 points
Items 4, 5, 8, 9	all 5 correct = 1 point; 4 correct = 0.5 points; 3,2,1,0 correct or missing = 0 points
Item 7	all 6 correct = 1 point; 5 correct = 0.5 points; 4,3,2,1,0 correct or missing = 0 points
Items 10 and 11	all 2 correct = 1 point; 1 correct: 0.5 point; 0 correct or missing: 0 points
Item 12	all 6 correct = 1 point; 5 or 4 correct = 0.5 points; 3,2,1,0 correct or missing = 0 points
Items 14 and 15	correct =1 point; not answered but circled correctly = 0.5 points; otherwise = 0 points

5 References

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